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Thermophoresis and chemical reaction effects on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge in the presence of suction or injection

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Abstract: An analysis is presented to investigate the effects of thermophoresis on non-Darcy MHD mixed convective heat and mass transfer of a viscous, incompressible and electrically conducting fluid past a porous wedge in the presence of chemical reaction. The wall of the wedge is embedded in a uniform non-Darcian porous medium in order to allow for possible fluid wall suction or injection. The governing boundary layer equations are written into a dimensionless form by similarity transformations. The transformed coupled nonlinear ordinary differential equations are solved numerically by using the R.K.Gill and shooting methods. Favorable comparison with previously published work is performed. Numerical results for the dimensionless velocity, temperature and concentration profiles as well as for the skin friction, heat and mass transfer and deposition rate are obtained and displayed graphically for pertinent parameters to show interesting aspects of the solution.

Key words: Chemical reaction, non-Darcy flow, Forchheimer number, mixed convection, thermophoresis and magnetic effect.

1. Introduction

Radiative heat and mass transfer flow is very important in manufacturing industries for the design of reliable equipment, nuclear plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. The effects of thermal radiation on the forced and free convection flows are also important in the context of space technology and processes involving high temperatures. In light of these various applications, England and Emery [1] studied the thermal radiation effect of an optically thin gray gas bounded by a stationary vertical plate. Raptis [2] studied radiation effect on the flow of a micro polar fluid past a continuously moving plate. Hossain and Takhar [3] analyzed the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Duwairi and Damesh [4, 5], Duwairi [6], Damesh et al. [7] studied the effect of radiation and heat transfer in different geometry for various flow conditions.

However, the phenomenon of thermophoresis plays a vital role in the mass transfer mechanism of several devices involving small micron sized particles and large temperature gradients in the fields. Thermophoresis principle is utilized to manufacture graded index silicon dioxide and germanium dioxide optical fiber performs used in the field of communications. Thermophoretic deposition of radioactive particles is considered to be one of the important factors causing accidents in nuclear reactors. Goldsmith and May [8] first studied the Thermophoretic transport involved in a simple one-dimensional flow for the measurement of the Thermophoretic velocity. Thermophoresis in laminar flow over a horizontal flat plate has been studied theoretically by Goren [8]. Shen [9] analyzed the problem of Thermophoretic deposition of small particles on to cold surfaces in two-dimensional and axi-symmetric cases. Thermophoresis in natural convection with variable properties for a laminar flow over a cold vertical flat plate has been studied by Jayaraj et al.[10]. Selim et al.[11] studied the effect of surface mass flux on mixed convective flow past a heated vertical flat permeable plate with thermophoresis.

Finally, thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and tower a cold one. Small particles, such as dust, when suspended in a gas with a temperature gradient, experience a force in the direction to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas particle trajectories from combustion devices, and studying the particulate material deposition turbine blades. The first analysis of thermophoretic deposition in geometry of engineering interest appears to be that of Hales et al.[12]. They have solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical surface situated adjacent to a large body of an otherwise quiescent air-steam-aerosol mixture. Simon [13] studied the effect of thermophoresis of aerosol particles in the laminar boundary layer on a flat plate. Recently, Chamkha and Pop [14] studied the effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium.

Transport processes in porous media play a significant roles in various applications such as in geothermal engineering, thermal insulation, energy conservation, petroleum industries solid matrix heat exchangers, chemical catalytic reactors, underground disposal of nuclear waste materials and many others. In many transport processes in nature and in industrial applications in which heat and mass transfer with variable viscosity is a consequence of buoyancy effects caused by diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies, such as polymer production and food processing. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Homogeneous reaction processes again can be classified into two such as (i) Destructive reaction ($k_1 > 0$) and (ii) Generative reaction ($k_1 < 0$) where k_1 is the rate of chemical reaction.

The effect of the presence of foreign mass on the free convection flow past a semi – infinite vertical plate was studied by Gebhart and Para [12]. The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated [13]. In most of cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself [14]. The effects of heat and mass transfer laminar boundary layer flow over a wedge have been studied by many authors [15 -25] in different situations. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been discussed by many authors [26-29] in various situations. The previous studies are based on the constant physical properties of the fluid. For most realistic fluids, the viscosity shows a rather pronounced variation with temperature. It is known that the fluid viscosity changes with temperature [30]. Then it is necessary to take into account the variation of viscosity with temperature in order to accurately predict the heat transfer rates. The effect of temperature-dependent viscosity on the mixed convection flow from vertical plate is investigated by several authors [30-33]. The effects of variable viscosity on non-Darcy mixed convection flow over a vertical surface have been studied by many authors [34-40] in different situations. Ali investigated the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface. Elbarbary and Elgazery examined Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces with radiation.

NOMENCLATURE

B_o	Magnetic induction,	u, v	velocity components in x and y direction.
U	Flow velocity away from the wedge	g	Acceleration due to gravity
k_1	Rate of chemical reaction.	K	Permeability of the porous medium
F	Empirical constant	T	Temperature of the fluid.
T_w	Temperature of the wall.	T_∞	Temperature far away from the wall
C	Species concentration of the fluid	C_w	Species concentration along the wall
C_∞	Species concentration away from the wall	c_p	specific heat at constant pressure
D	effective diffusion coefficient	V_T	thermophoretic velocity
ρ	Density of the fluid	α_m	thermal conductivity of the fluid
σ	Electric conductivity of the fluid	α	Thermal diffusivity
ν	kinematic viscosity	μ	Dynamic viscosity
β	Coefficient of thermal expansion	β^*	Concentration expansion coefficients

The aim of this work is to study the effects of variable viscosity and thermophoresis on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. The order of chemical reaction in this work is taken as first-order reaction. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. MATHEMATICAL ANALYSIS

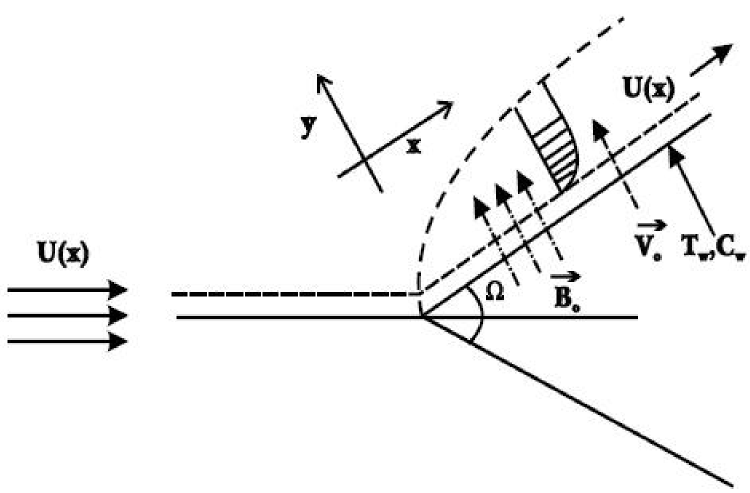

Fig.1 Flow analysis along the wall of the wedge

Let us consider a steady, laminar, hydro magnetic coupled heat and mass transfer by mixed convection flow in front of a stagnation point on a wedge plate embedded in porous medium. The fluid is assumed to be Newtonian, electrically conducting and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall, C_∞, is infinitesimally small [13]. Let x-axis be taken along the direction of the wedge and y-axis normal to it. A uniform transverse magnetic field of strength B_o is applied parallel to the y-axis. The chemical reaction is taking place in the flow and the effects of thermophoresis are being taken into to help in the understanding of the mass deposition variation on the surface. Fluid suction or injection is imposed at the wedge surface, see Fig.1. Due to the boundary layer behavior the temperature gradient in the y-direction is much larger than that in the x-direction and hence only the thermophoretic velocity component which is normal to the surface is of importance. It is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion for mixed convection under Boussinesq's approximation including variable viscosity are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

(2)
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+U\frac{dU}{dx}-\frac{\sigma B_0^2}{\rho}(u-U)-\frac{\nu}{K}(u-U)$$
$$-\frac{F}{\sqrt{K}}(u^2-U^2)+[g\beta(T-T_\infty)+g\beta^*(C-C_\infty)]\sin\frac{\Omega}{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha_m}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\left(\frac{\mu}{\rho c_p}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho c_p}\left(u^2-U^2\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial\left(V_T C\right)}{\partial y}-k_1 C \tag{4}$$

The boundary conditions are,

$$u=0,\ v=-v_0,\ T=T_w,\ C=C_w \text{ at } y=0 \tag{5}$$

$$u=U(x),\ T=T_\infty,\ C=C_\infty \text{ at } y\longrightarrow\infty \tag{6}$$

where u, v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, g is the acceleration due to gravity, σ is the electrical conductivity, ρ is the density of the fluid, β is the coefficient thermal expansion, $\beta*$ is the coefficient of volumetric expansion, T, T_w, $T_\infty$are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while C, C_w, C_∞ are the corresponding concentrations, B_o is the magnetic induction, α_m is the thermal

conductivity of the fluid, μ is the dynamic viscosity, D is the effective diffusion coefficient, c_p is the specific heat at constant pressure, $k_1$is the rate chemical reaction, K is the permeability of the porous medium, Ω is the angle of inclination of wedge, V_T is the thermophoretic velocity and F is the empirical constant (Forchheimer number) in the second order resistance and setting F = 0 in Equ.(2) is reduced to the Darcy law [48]. The fourth and fifth terms on the right-hand side of Equ.(2) stand for the first-order (Darcy) resistance and second-order (porous inertia) resistance, respectively. The second term on the right-hand side of Equ.(3) represents viscous dissipation and the last term indicates the Ohmic heating effect.
Following the lines of Kafoussias et al. [13], the following change of variables are introduced

$$\eta(x,y) = y\sqrt{\frac{(1+m)U}{2\nu\, x}} \tag{7}$$

$$\psi(x,y) = \sqrt{\frac{2U\nu\, x}{1+m}} f(x,\eta). \tag{8}$$

The viscosity is assumed to be an inverse linear function of temperature given by the following [24]

$$\frac{1}{\mu} = \frac{1}{\mu_a}[1+\chi(T-T_a] \tag{9}$$

where $\mu_a$is the ambient fluid dynamic viscosity and χ is a thermal property of the fluid.
Equ.(9) can be written as follows

$$\frac{1}{\mu} = a(T-T_r] \tag{10}$$

where $a = \frac{\chi}{\mu_a}$ and $T_r = T_a - \frac{1}{\chi}$ are constants and their values depend on the reference state and the thermal property of the fluid.
Under this consideration, the potential flow velocity can be written as

$$U(x) = Ax^m,\ \beta_1 = \frac{2m}{1+m} \tag{11}$$

where A is a constant and β_1 is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \frac{\Omega}{\pi}$ for a total angle Ω of the wedge.
The continuity equation (1) is satisfied by the stream function $\psi(x,y)$ defined by

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{12}$$

Transform the Equs. (2), (3) and (4) into a set of ordinary differential equations, we introduce the following dimensionless parameters and variables,

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{14}$$

(15) $Gr_1 = \frac{\nu g \beta (T_w - T_\infty)}{U^3}$ (Grash of number)

(16) $Gc_1 = \frac{\nu g \beta * (C_w - C_\infty)}{U^3}$ (mass transfer Grash of number)

(17) $Ec = \frac{c^2}{c_p (T_w - T_\infty)} (k^2)^{\frac{2m}{1-m}}$ (Eckert number)

(18) $N = \frac{\beta^* (C_w - C_\infty)}{\beta (T_w - T_\infty)}$ (Buoyancy ratio)

(19) $Re_x = \frac{Ux}{\nu}$ (Reynolds number)

(20) $Re_k = \frac{U\sqrt{K}}{\nu}$ (Modified local Reynolds number)

(21) $\Pr = \frac{\nu}{\alpha_m}$ (Prandtl number)

(22) $Fn = \frac{FU\sqrt{K}}{\nu}$ (Forchheimer number)

(23) $Sc = \frac{\nu}{D}$ (Schmidt number)

(24) $M^2 = \frac{\sigma B_0^2}{\rho A}$ (magnetic parameter)

(25) $S = v_0\sqrt{\frac{(1+m)x}{2\nu U}}$ (suction or injection parameter)

(26) $\gamma = \frac{\nu k_1}{U^2}$ (chemical reaction parameter)

(27) $\lambda = \frac{\alpha_m}{KU}$ (Porous medium parameter)

(27a) $\tau = -\frac{k(T_w - T_\infty)}{T_r}$ (Thermophoretic parameter)

Now the equations (2) to (4)

$$(\theta - \theta_r)\frac{\partial^3 f}{\partial \eta^3} = \frac{(\theta-\theta_r)^2}{\theta_r}[-f\frac{\partial^2 f}{\partial \eta^2} - \frac{2m}{1+m}(1-(\frac{\partial f}{\partial \eta})^2) - \frac{2}{1+m}\frac{N\phi+\theta}{1+N}GrRe_x \sin\frac{\Omega}{2} + \frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}) + \frac{2x}{m+1}\frac{\sigma B_0^2}{\rho U}(\frac{\partial f}{\partial \eta} - 1) + \frac{2}{m+1}\lambda(\frac{\partial f}{\partial \eta} - 1) + \frac{2}{m+1}\frac{Fx}{\sqrt{K}}((\frac{\partial f}{\partial \eta})^2 - 1)] + \frac{\partial \theta}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2} \tag{28}$$

(29)

$$\frac{\partial^2 \theta}{\partial \eta^2} = -\Pr\frac{\partial \theta}{\partial \eta} + \frac{2\Pr}{1+m}(\theta + \frac{n}{1-n})\frac{\partial f}{\partial \eta} + \Pr\frac{2x}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial \eta}) - \frac{2}{1+m}\Pr Ec(\frac{\partial^2 f}{\partial y^2})^2 - \frac{2\Pr}{1+m}\frac{\sigma B_0^2}{\rho U}\frac{U^2}{c_p(T_w - T_\infty)}((\frac{\partial f}{\partial y})^2 - 1)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} = -Sc\, f\frac{\partial \phi}{\partial \eta} + \frac{2\, Sc\, x}{1+m}\gamma\, \phi + \frac{2\, Sc}{1+m}\phi\frac{\partial f}{\partial \eta} + \frac{2xSc}{1+m}(\frac{\partial f}{\partial \eta}\frac{\partial \phi}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \phi}{\partial \eta}) \tag{30}$$

The boundary conditions can be written as

$$\eta = 0 : \frac{\partial f}{\partial \eta} = 0,\ \frac{f}{2}(1 + \frac{x}{U}\frac{dU}{dx}) + x\frac{\partial f}{\partial x} = -v_0\sqrt{\frac{(1+m)x}{2\nu U}}\ \theta = 1,\ \phi = 1$$

$$\eta \longrightarrow \infty : \frac{\partial f}{\partial \eta}1,\ \theta = 0,\ \phi = 0 \tag{31}$$

where $v_0$is the velocity of suction if $v_0 < 0$ and injection if $v_0 > 0$ and Gr = $Gr_1 + Gc_1$.

The equations (28) to (30) and boundary conditions (31) can be written as follows:

(32)

$$\frac{\partial^3 f}{\partial\eta^3}+\frac{(\theta-\theta_r)}{\theta_r}[(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial\xi})\frac{\partial^2 f}{\partial\eta^2}-\frac{1-m}{1+m}\xi\frac{\partial^2 f}{\partial\xi\,\partial\eta}\frac{\partial f}{\partial\eta}-\frac{2}{1+m}M^2\xi^2(\frac{\partial f}{\partial\eta}-1)$$
$$+\frac{2}{1+m}\frac{N\phi+\theta}{1+N}GrRe_x\sin\frac{\Omega}{2}-\frac{2}{m+1}\xi^2\lambda\Pr(\frac{\partial f}{\partial\eta}-1)$$
$$-\frac{2}{m+1}((\frac{\partial f}{\partial\eta})^2-1)(\frac{Rex}{(Rek)^2}Fn+m)]-\frac{2}{1+m}\frac{1}{\theta-\theta_r}\frac{\partial\theta}{\partial\eta}\frac{\partial^2 f}{\partial\eta^2}=0$$

(33)

$$\frac{\partial^2\theta}{\partial\eta^2}+\Pr(f+\frac{1-m}{1+m}\xi\frac{\partial f}{\partial\xi})\frac{\partial\theta}{\partial\eta}-\frac{2\Pr}{1+m}(\theta+\frac{n}{1-n})\frac{\partial f}{\partial\eta}-\frac{1-m}{1+m}\xi\frac{\partial\theta}{\partial\xi}\frac{\partial f}{\partial\eta}+\frac{2}{1+m}\Pr Ec(\frac{\partial^2 f}{\partial y^2})^2$$
$$+\frac{2\Pr}{1+m}M^2Ec\xi^{\frac{2(1+m)}{1-m}}((\frac{\partial f}{\partial y})^2-1)=0$$

(34)

$$\frac{\partial^2\phi}{\partial\eta^2}+Sc\,f\,\frac{\partial\phi}{\partial\eta}-\frac{2Sc}{1+m}\xi^2\gamma\,\phi+Sc\frac{1+m}{1-m}(\frac{\partial\phi}{\partial\eta}\xi\frac{\partial f}{\partial\xi}-\frac{\partial f}{\partial\eta}\xi\frac{\partial\phi}{\partial\xi})-\frac{2Sc}{1+m}\phi\frac{\partial f}{\partial\eta}=0$$

$$\eta=0:\frac{\partial f}{\partial\eta}=0,\ \frac{(1+m)f}{2}+\frac{1-m}{2}\xi\frac{\partial f}{\partial\xi}=-S,\ \theta=1,\ \phi=1$$

(35)
$$\eta\longrightarrow\infty:\frac{\partial f}{\partial\eta}=1,\ \theta=0,\ \phi=0$$

where S is the suction parameter if $S>0$ and injection if $S<0$, Fn is the dimensionless inertial parameter (Forchheimer number) and $\xi=kx^{\frac{1-m}{2}}$ [13], is the dimensionless distance along the wedge ($\xi>0$). In this system of equations, $f(\xi,\eta)$ is the dimensionless stream function; $\theta(\xi,\eta)$ is the dimensionless temperature; $\phi(\xi,\eta)$ is the dimensionless concentration; Pr is the Prandtl number, Re_x is the Reynolds number etc. which are defined in (13) to (27). The parameter ξ indicates the dimensionless distance along the wedge ($\xi>0$).It is obvious that to retain the $\xi-$derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wise locations through the $\xi-$derivatives, a locally autonomous solution, at any given stream wise location, can not be obtained. In such a case, an implicit marching numerical solution scheme is usually applied preceding the solution in the $\xi-$direction, i.e. calculating unknown profiles at ξ_{l+1} when the same profiles at ξ_l are known. The process starts at $\xi=0$ and the solution proceeds from ξ_l to ξ_{l+1} but such a procedure is time consuming.
However, when the terms involving $\frac{\partial f}{\partial\xi},\frac{\partial\theta}{\partial\xi}$ and $\frac{\partial\phi}{\partial\xi}$and their η derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions f, θ and ϕ with ξ as a parameter and the computational task is simplified. Furthermore a locally autonomous

solution for any given ξ can be obtained because the stream wise coupling is severed. So, following the lines of [13], R.K.Gill and shooting numerical solution scheme are utilized for obtaining the solution of the problem. Now, due to the above mentioned factors and for $\xi = 1.0$, the equations (31) to (33) are changed to

(36)
$$f'''+\frac{\theta-\theta_r}{\theta_r} f\, f''+\frac{2}{1+m}\frac{\theta-\theta_r}{\theta_r}(1-f'^2)\left(\frac{Rex}{(Rek)^2}Fn+m\right)+\frac{2}{1+m}\frac{\theta-\theta_r}{\theta_r}\frac{N\phi+\theta}{1+N}GrRe_x\sin\frac{\Omega}{2} -\frac{\theta-\theta_r}{\theta_r}\frac{2}{1+m}(M^2+\Pr\lambda)(f'-1)-\frac{2}{1+m}\frac{1}{\theta-\theta_r}\theta'\, f''=0$$

(37)
$$\theta''+\Pr f\,\theta'-\frac{2\Pr}{1+m}\left(\theta+\frac{n}{1-n}\right)f'+\frac{2}{1+m}\Pr Ecf''^2+\frac{2\Pr}{1+m}M^2Ec\,(f'^2-1)=0$$

(38)
$$\phi''+Sc\,f\,\phi'-\frac{2Sc}{1+m}f'\phi-\frac{2\,Sc}{1+m}\gamma\,\phi=0$$

with boundary conditions

$$\eta=0: f(0)=-\frac{2}{1+m}S,\ f'(0)=0,\ \theta(0)=1,\ \phi(0)=1$$

(39)
$$\eta\longrightarrow\infty: gf'(\infty)=1,\ \theta(\infty)=0,\ \phi(\infty)=0$$

3. NUMERICAL SOLUTION

The set of non-linear ordinary differential equations (36) to (38) with boundary conditions (39) have been solved by using the R .K Gill method along with shooting technique with $\alpha, \Omega, M2$ and n as prescribed parameters. The computation were done by a program which uses a symbolic and computational computer language Matlab. A step size of $\Delta\eta$= 0.001 was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of η_∞ was found to each iteration loop by assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters α, Ω, M^2 and n determined when the values of unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7}. Effects of variable viscosity and thermal stratification on heat and mass transfer on non-Darcy MHD mixed convection flow over a porous wedge with Ohmic heating are studied for different values of suction / injection at the wall of the wedge and the strength of the chemical reaction. In the following section, the results are discussed in detail.

4. RESULTS AND DISCUSSION

The computations have been carried out for various values of variable viscosity $\theta_r = Qr$, chemical reaction(γ), magnetic parameter (M^2), Forchheimer number (Fn), thermal stratification (n) and porous medium (λ). In order to validate our method, we have compared steady state results of skin friction, $f''(0)$ and rate of heat transfer $-\theta'(0)$ for various values of θ_r (Table.1) with those of and found them in excellent agreement.

	Pantokratoras,[51]		Present work		
θ_r	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	Pr
2.0	0.2642	-0.7341	0.2651	-0.7345	20.00
4.0	0.3785	-0.7888	0.3791	-0.78921	13.33
6.0	0.4145	-0.8042	0.4167	-0.8061	12.00
8.0	0.4322	-0.8112	0.4341	-0.8127	11.43
10	0.4426	-0.8162	0.4452	-0.8171	11.11

Table.1: Comparison of the values of $f\,''(0)$ $\alpha\nu\delta$ $-\theta'(0)$ for various values of θ_r with $\lambda = 0$, $\Omega = 30°$, $N = 0$, $m = 0.0909$, $Sc = 0$, $M^2 = n = 0$ and $\gamma = S = Fn = 0$.

The velocity, temperature and concentration profiles obtained in the dimensionless form are presented in Figure.2-8 for Pr = 0.71 which represents air at temperature 20 ^{0}C and Sc = 0.62 which corresponds to water vapor that represents a diffusion chemical species of most common interest in air. Grashof number for heat transfer is chosen to be Gr = 4.0, since these values corresponds to a cooling problem, Eckert number, Ec = 0.001, buoyancy ratio, $N = 1.0$, local Reynolds number, $Rek = 1.0$, constant, m = 0.0909 (for angle of inclination of wedge, $\beta_1 = \frac{\Omega}{\pi} = \frac{2m}{1+m}$), porous medium parameter, $\lambda = 1.0$, and Reynolds number Rex = 3.0. The values of γ are chosen to be 0.5, 1.0 and 2.5. It is important to note that θ_r is negative for liquids and positive for gases when $T_w - T_\infty$ is positive. The value of θ_r (for air θ_r>0) are chosen to be 0.5, 1.0 and 3.0 and the value of suction, S is chosen to be 5.0. The values of strength of the magnetic field are chosen to be 0.5, 2.0 and 6.0 and thermal stratification is chosen to be 0.1, 0.5 and 0.9.

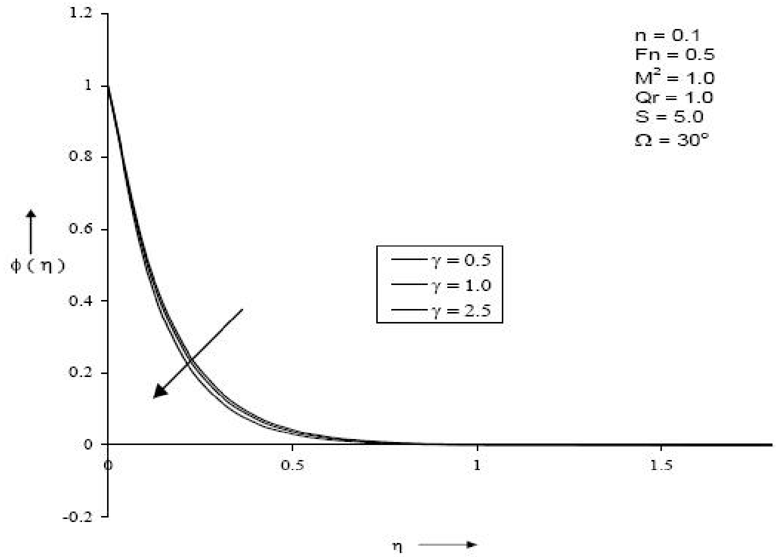

Fig.2: Effects of chmical reaction over the concentration profiles

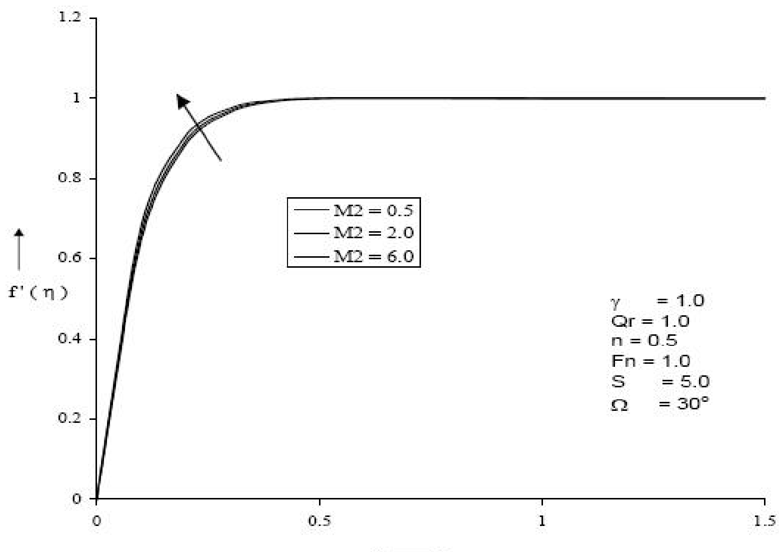

Fig.3: Influence of magnetic field over the velocity profiles

Effects of the chemical reaction γ on concentration profiles are shown in Fig.2. It is seen from this figure that the concentration of the fluid decreases with increase of chemical reaction, while the profiles for velocity and temperature are not significant with increase of chemical reaction.

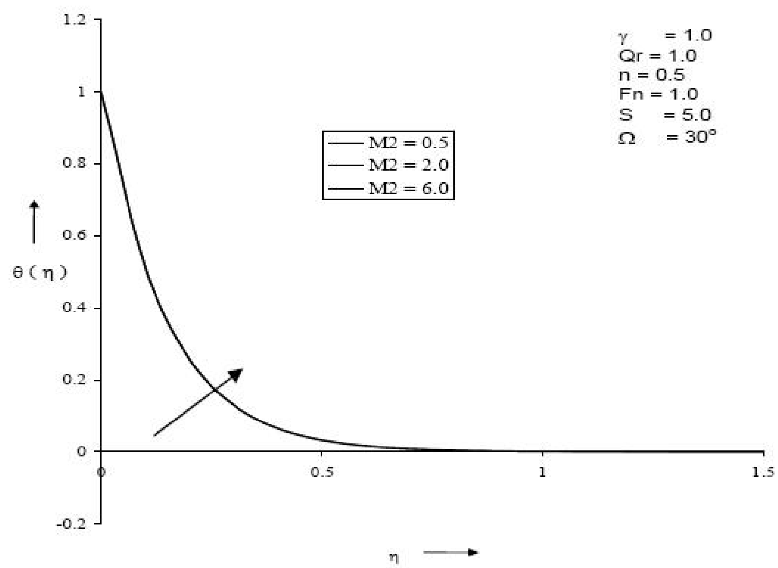

Fig.4: Effects of magnetic field over the temperature profiles

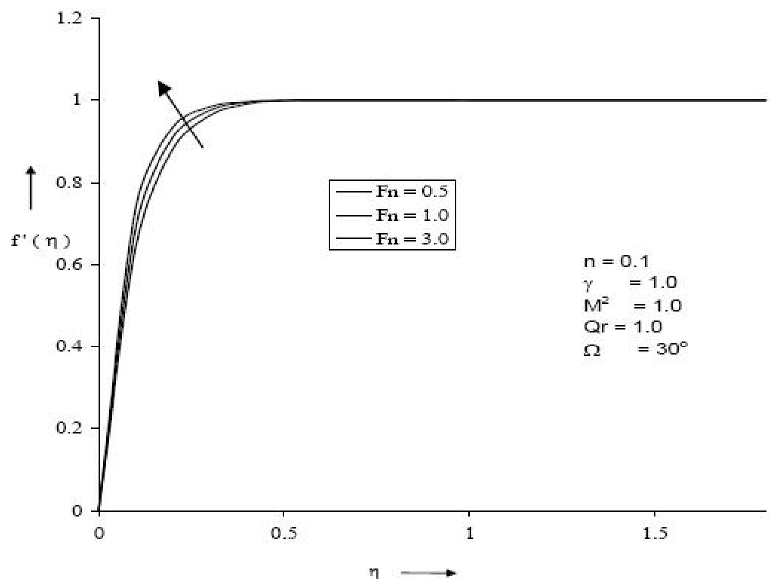

Fig.5: Effects of Forchheimer nmber over the velocity profiles

Effects of magnetic field on velocity and temperature profiles are shown through Figs.3 and 4. It is seen from these figures that the velocity of the fluid increases and the temperature of the fluid slightly increase with increase of the strength of magnetic field. As the strength of the magnetic effect increases, the Lorentz force, which opposes to the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations since magnetic field exerts retarding force on the mixed convection flow.

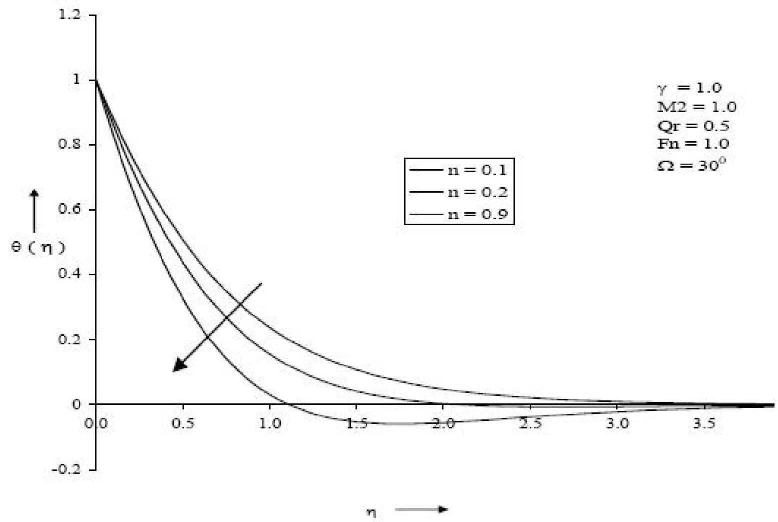

Fig.6: Thermal stratification over the temperature profiles

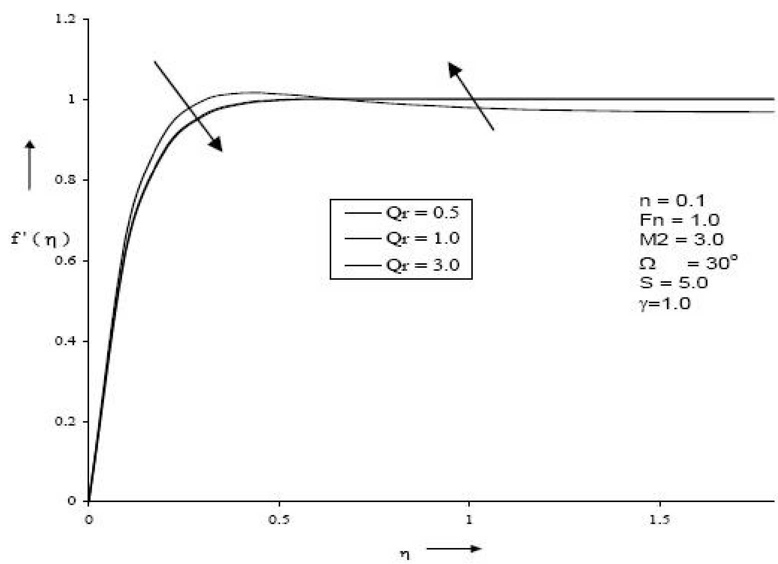

Fig.7: Influence of viscosity over the velocity profiles

Figure 5 shows the influence of the inertial parameter Fn (Forchheimer number) on the dimensionless velocity profiles. It is obvious that the velocity decreases dramatically as the inertial parameter, Fn increases. The reason for this behavior is that the inertia of the porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with raised temperature. These behaviors are shown in Fig.5, while the temperature and concentration profiles are not significant with increase of Forchheimer number.

The dimensionless velocity profiles for different values of thermal stratification are plotted in Fig.6. Due to the uniform viscosity, $\theta_r = 0.5$, it is clear that the temperature of the fluid decreases with increase of thermal stratification while the velocity and concentration are not significant with increase of thermal stratification effects. In particular, the temperature of the fluid gradually changes from higher value to the lower value only when the strength of the thermal stratification is higher than the viscosity parameter. For heat transfer characteristics mechanism, interesting result is the large distortion of the temperature field caused for n = 0.9. Negative value of the temperature profile is seen in the outer boundary region for n = 0.9 and $\theta_r = 0.5$. All these physical behaviors are due to the combined effects of the strength of the viscosity and magnetic effects.

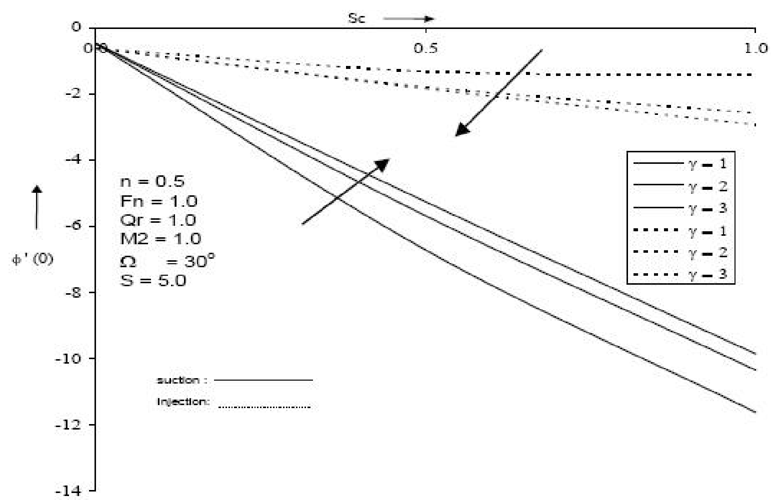

Fig.8: Chemical reaction over the rate of mass tranfor

Fig.7 depicts the dimensionless velocity profiles f '(η) for different values of viscosity. Due to the uniform suction with fixed angle of inclination of the wall of the wedge, the effect of increase of viscosity is to decrease as well as to increase the velocity component of the fluid along the wall of the wedge, which is depicted through Fig.7, while the temperature and concentration are not significant with increase of viscosity. So, it is also observed that the velocity of the fluid gradually changes from higher value to the lower value and lower value to higher value only when the diffusive effect D is smaller than kinematics viscosity. All these physical behavior are due to the combined effects of heat

source and viscosity effects at the wall of the wedge.
Effects of the strength of chemical reaction on the rate mass transfer are shown in Figure 8. In the presence of suction, it is seen that the rate of mass transfer increases and for injection, the rate of mass transfer decreases with increase of chemical reaction. So, in the case of suction, it is interesting to note that increasing the chemical reaction effect is to decrease the concentration in the boundary layer and thus increase the rate of mass transfer at wall, but the opposite trend is true when the chemical reaction strength is increased. All these physical behavior are due to the combined effect of the strength of chemical reaction and viscosity at the wall of the wedge.

5. CONCLUSIONS

This paper studied the effects of variable viscosity and thermal stratification on non-Darcy MHD mixed convective heat and mass transfer past a porous wedge with Ohmic heating in the presence of chemical reaction. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters. Comparisons with previously published works are performed and excellent agreement between the results is obtained.
We conclude the following from the results and discussions

♦ Velocity of the fluid increases and the temperature of the fluid slightly increases with increase of the strength of magnetic field. As the strength of the magnetic effect increases, the Lorentz force, which oppose the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations since magnetic field exerts retarding force on the mixed convection flow.

♦ In the presence of uniform chemical reaction, velocity decreases dramatically as the inertial parameter, Fn increases. The reason for this behavior is that the inertia of the porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with raised temperature.

♦ Temperature of the fluid gradually changes from higher value to the lower value only when the strength of the thermal stratification is higher than the viscosity parameter. For large heat transfer characteristics mechanism, interesting result is the large distortion of the temperature field caused for n = 0.9. Negative value of the temperature profile is seen in the outer boundary region for n = 0.9 and $\theta_r = 0.5$. All these physical behavior are due to the combined effects of the strength of the viscosity and magnetic effects.

♦ Due to the uniform suction with fixed angle of inclination of the wall of the wedge, the effect of increase of viscosity is to decrease as well as to increase the velocity component of the fluid along the wall of the wedge, which is depicted through Fig.7, while the temperature and concentration are not significant with increase of viscosity. So, it is also observed that the velocity of the fluid gradually changes from higher value to the lower value and lower value to higher value only when the diffusive effect D is smaller than kinematics viscosity. All these physical behavior are due to the combined effects of heat source and viscosity effects at the wall of the wedge.

⧫ In the case of suction, it is interesting to note that increasing the chemical reaction effect is to decrease the concentration in the boundary layer and thus increase the rate of mass transfer at wall, but the opposite trend is true when the chemical reaction strength is increased. All these physical behavior are due to the combined effect of the strength of chemical reaction and viscosity at the wall of the wedge.

It is hoped that the present investigation may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field in the migration of underground water and in the filtration and water purification processes. The results of the problem are also of great interest in geophysics in the study of interaction of the geomagnetic field with the fluid in the geothermal region.

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REFERENCES

1.Gehart,B., and Pera,L.,1971, The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, International Journal of Heat Mass Transfer, 14, 2025.

2. Byron Bird,R; Warren E.Stewart.; Edwin N. Lightfoot., 1992, Transport phenomena, John Wiley and sons, New York.

3. Cussler, E.L., 1988, Diffusion Mass Transfer in Fluid Systems, Cambridge University Press, London, UK.

4. Yih,K.A., 1998, Uniform suction / blowing effect on force convection about wedge, Acta Mech., 128,173.

5. Kafoussias, N.G.; Nanousis,N.D.,1997, Magnetohydrodynamiclaminar boundary layer flow over a wedge with suction or injection. Can. Journal of Physics, 75,733.

6.Kumari, M.,1998, Effect of large blowing rates on the steady laminar incompressible electrically conducting fluid over an infinite wedge with a magnetic field applied parallel to the wedge, International Journal of Engng.Sci., 36, 299.

7. Anjali Devi, S.P. and Kandasamy.R.,2001, Effects of heat and mass transfer on MHD laminar boundary layer flow over a wedge with suction or injection, Journal of Energy,Heat and Mass Transfer, 23, 167.

8. Yih, K.A., 1998, The effect of uniform suction/blowing on heat transfer of MHD Hiemenz flow through porous media, Acta Mech., 130,147.

9. Watanabe,T., 1990, Thermal boundary layer over a wedge with uniform suction or injection in forced flow, Acta Mechanica, 83,119.

10. Chamkha,A.J.; Khaled,A.R.A.,2001, Similarity solutions for hydro magnetic simultaneous heat and mass transfer, Heat Mass Transfer, 37,117.

11. Hossian,M.A., 1992, Viscous and Joule heating effects on MHD free convection flow with variable plate temperature, Int. journal of heat and mass transfer, 35, 3485.
12. Hakiem,M.A.EL.; Mohammadeian, A.A.;Kaheir, S.M.M.EL.Gorla, R.S.R.,1999, Joule heating effects on MHD free convection flow of a micro polar fluid, International Comms. Heat Mass Transfer, 26,219.
13. Kuo Bor-Lih., 2005, Heat transfer analysis for the Falkner-Skan wedge flow by the differential transformation method, Int.al J.of Heat Mass Transfer, 48,5036.
14. Cheng, W.T., Lin, H.T.,2002, Non-similarity solution and correlation of transient heat transfer in laminar boundary layer flow over a wedge, Int. J.of Engg.Sci., 40,531.
15. Apelblat, A., 1982, Mass transfer with a chemical reaction of the first order. Effects of axial diffusion, The chemical Engineering Journal, 23,193.
16. Das,U.N., Deka, R and Soundalgekar, 1994, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen, 60, 284.
17. Muthucumaraswamy, R. and Ganesan,P.,2001, Effects of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate, Journal of Applied Mechanics and Technical Physics, 42, 665
18. Kandasamy,R., Periasamy, K., and Sivagnana Prabhu,K.K.,2005, Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection, International Journal of Heat and Mass Transfer, 48, 1388.
19. Herwing,H. and Wickern, 1986, The effects of variable properties on laminar boundary layer flow, Warme-und Stoffiibctragung, 20,47.
20. Lai, E.c. and Kulacki, F.A., 1990, The effects of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, International Journal of Heat and Mass Transfer, 33, 1028.
21. Kafoussias, N.g. and Williams, E.w., 1995, The effects of temperature-dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal flat plate, Acta Mechanica, 110, 123.
22. Hady,E.M., Bakier, A.Y., and Gorla, R.S.R.,1996, Mixed convection boundary layer flow on a continuous flat plate with variable viscosity, Heat and Mass Transfer, 31, 169.
23. S.Gill.,1951, Proceeding of Cambridge Philosophical Society, pp.96-123.
24. Pantokratoras,A.,2006, The Falkner- Skan flow with constant wall temperature and variable viscosity, International Journal of Thermal Sciences, 45,378.
25. Chien-Hsin Chen, 1997, Non-Darcy mixed convection over a vertical flat plate in porous media with variable wall heat flux, International communication Heat Mass Transfer,24,No., pp.427-437,
26. K.A. Yih, 1998, Blowing/suction effect on non-Darcy forced convection flow about a flat plate with variable wall temperature in porous media, Acta Mechanica,131, pp.255-265.
27. Ahmed A. Afify, (2007) Effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium, Applied Mathematical Modeling,31, pp.1621- 1634.
28. Ali J. Chamkha,(1996), Non-Darcy hydromagnetic free convection from a cone

and a wedge in porous media, International Communications in Heat and Mass Transfer, 23, pp.875-887.
29. S. Jayanthi and M. Kumari, (2007), Effect of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a non-Newtonian fluid saturated porous medium, Applied Mathematics and Computation,186, pp.1643-1659.
30. Ching-Yang Cheng, (2006), Non-Darcy natural convection heat and mass transfer from a vertical wavy surface in saturated porous media, Applied Mathematics and Computation,182, pp.1488-1500.
31. S. Jayanthi and M. Kumari, (2006), Effect of variable viscosity on non-Darcy free or mixed convection flow on a vertical surface in a fluid saturated porous medium, Mechanics Research Communication,33, pp.148-156.
32. B. V. Rathish Kumar and Shalini, (2004), Darcy free convection induced by a vertical wavy surface in a thermally stratified porous medium, International Journal of Heat Mass Transfer,47, 2353-2363.
33. M. Kumari and G. Nath, Non-Darcy mixed convection in power-law fluids along a non-isothermal horizontal surface in a porous medium, International Journal of Engineering Science, 42, pp.353-369.
34. K. A. Yih , (2000), Viscous and Joule heating effects on non-Darcy MHD natural convection flow over a permeable sphere in porous media with internal heat generation, International Communications in Heat and Mass Transfer, 27, pp.591-600.
35. M. A. Hossain, Kambiz Vafai and Khalil M. N. Khanafer, (1999), Non-Darcy natural convection heat and mass transfer along a vertical permeable cylinder embedded in a porous medium, International Journal of Thermal Sciences,38, pp.854-862.
36. M.Q. AI-Odat, F.M.S. AI-Hussien and R.A.Damesh, (2005), Influence of radiation on mixed convection over a wedge in non-Darcy porous medium, Forsch Ingenieurwes,69, pp.209-215.
37. Mohamed E. Ali, 2006, The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface, International journal of Thermal Sciences,45, pp.60-69.
38. Elsayed M.E. Elbarbary, Nasser S. Elgazery, 2004,Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces with radiation, International Journal of Thermal Sciences, 43, pp. 889-899.
39. Bejan,A.(1995),Convection heat transfer,p.520,New Yark: Wiley-Interscience.
40. Nakayama, A. and Koyama, H.,(1989), Similarity solutions for buoyancy induced flows over a non isothermal curved surface in thermally stratified porous medium, Applied Scientific Research, 46 ,pp.309-322.