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Maximum Likelihood Estimation and Confidence Intervals of System Reliability for Gompertz Distribution in Stress-Strength Models[1]

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Summary. A stress-strength model defines life of a component which has strength X and is subjected to stress Y. In this paper, we consider the estimation problem of $R = P(Y < X)$ when X ˜ $Gompertz(c, \lambda_1)$ and Y ˜ $Gompertz(c, \lambda_2)$ are independent with c known. R can be considered to be the reliability of a system and is known to be stress-strength reliability. The maximum likelihood estimate of R is derived and various distributional properties of this estimator is discussed. Exact and asymptotic confidence intervals for R are constructed. Also a simulation study is performed to investigate the coverage probabilities of these intervals.

Key words: Coverage probability; Stress-strength reliability; Gompertz distribution; Minimum variance unbiased estimation; Maximum likelihood estimation; Confidence interval

1. Introduction

The term "stress-strength reliability" in statistical literature typically refers to the quantity $P(Y < X)$. This term states the reliability of a system of strength X subjected to a stress Y. The system fails if the applied stress exceeds its strength. Thus the quantity is known to be the stress-strength reliability of the system and is typically denoted by R . In other words, the stress-strength reliability of the system is the probability that the system is strong enough to overcome the stress imposed on it. The problem arises in some fields, for example, in biometry, Y represents a patient's remaining years of life if treated with drug A and X represents the patient's remaining years of life if treated

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with drug B. If the choice is left to the patient, person's deliberations will center on whether $P(Y < X)$ is less than or greater than 1/2 (Ali and Woo, 2005a, 2005b).

Some authors have considered different choices for stress and strength distributions. The stress-strength reliability and it's estimation problems for several distributions are discussed in the works of Church and Harris (1970) , Downton (1973), Woodward and Kelley (1977), for the family of normal distributions, Tong (1974, 1975a, 1975b), Sathe and Shah (1981), Chao (1982) for the family of exponential distributions, Beg and Singh (1979) for the family of pareto distributions, Awad and Gharraf (1986) for the family of Burr XII distributions, Constantine et al. (1986), and Ismail et al. (1986) for the family of gamma distributions, McCool (1991), Kundu and Gupta (2006) for the family of weibull distributions, Surles and Padgett (1998, 2001), Raqab and Kundu (2005) for the family of burr X distributions, Ali and Woo (2005a, 2005b) for the family of levy and p-dimensional rayleigh distributions,, Kundu and Gupta (2005) for the family of generalized exponential distributions and Mokhlis (2005) for the family of burr III distributions. Recently, Kotz et al. (2003) have presented a review of all methods and results on the stress-strength model in the last four decades.

This paper is organized as follows; In Section 2, the stress-strength reliability is derived underlying The *Gompertz* distribution. In Section 3, maximum likelihood estimate (MLE) of the stress-strength reliability (R) is obtained and various distributional properties of this estimator is discussed. Also, mean squares error (MSE) of the these estimates are compared. In Section 4, exact and asymptotic confidence intervals for the stress-strength reliability are constructed and a simulation study is performed to investigate the coverage probabilities of these intervals as well.

2. Stress-Strength Reliability

Let X be the strength of a system and Y be the stress acting on it. X and Y are the random variables from *Gompertz* with parameters (c_1, λ_1) and (c_2, λ_2) respectively. That is, the probability density functions and the cumulative distribution functions of X and Y are, respectively

$$(2.1)\quad f_X(x) = \lambda_1 \exp(c_1 x) \exp\left\{-\lambda_1 c_1^{-1} \left[\exp(c_1 x) - 1\right]\right\}, x > 0, c_1 > 0, \lambda_1 > 0$$

$$(2.2)\quad F(x) = 1 - \exp\left\{-\lambda_1 c_1^{-1} \left[\exp(c_1 x) - 1\right]\right\}$$

and

$$(2.3)\quad f_Y(y) = \lambda_2 \exp(c_2 y) \exp\left\{-\lambda_2 c_2^{-1} \left[\exp(c_2 y) - 1\right]\right\}, y > 0, c_2 > 0, \lambda_2 > 0$$

(2.4) $F(y) = 1 - \exp\left\{-\lambda_2 c_2^{-1}\left[\exp(c_2 y) - 1\right]\right\}$

where c_1 and c_2 are known parameters and also λ_1 and λ_2 are unknown parameters. Then R is

$$
\begin{aligned}
R = P(Y < X) &= \int_0^{\infty} P(Y < x) f_X(x)\, dx \\
&= \int_0^{\infty} \left[1 - \exp\left\{-\frac{\lambda_2}{c_2}(e^{c_2 x} - 1)\right\}\right] \lambda_1 e^{c_1 x} \exp\left\{-\frac{\lambda_1}{c_1}(e^{c_1 x} - 1)\right\} dx \\
&= 1 - \exp\left\{\frac{\lambda_1}{c_1} + \frac{\lambda_2}{c_2}\right\} \int_{\lambda_1/c_1}^{\infty} \exp\left\{-\frac{\lambda_2}{c_2}\left(\frac{c_1 t}{\lambda_1}\right)^{c_2/c_1}\right\} e^{-t} dt
\end{aligned} \tag{2.5}
$$

If we write the identity given by Eq.(2.6) in the right hand side of the integral given by Eq.(2.5)

(2.6) $\exp\left\{-\frac{\lambda_2}{c_2}\left(\frac{c_1 t}{\lambda_1}\right)^{c_2/c_1}\right\} = \sum_{k=0}^{\infty} \frac{(-1)^k (\lambda_2/c_2)^k (tc_1/\lambda_1)^{kc_2/c_1}}{k!}.$

The final form of Eq.(2.5) is rearranged as follows;

$$
\begin{aligned}
R = 1 - \exp&\left\{\frac{\lambda_1}{c_1} + \frac{\lambda_2}{c_2}\right\} \\
&\times \sum_{k=0}^{\infty} \frac{(-1)^k (\lambda_2/c_2)^k (c_1/\lambda_1)^{kc_2/c_1}}{k!} \\
&\quad \times \left[\Gamma(kc_2/c_1 + 1) - \int_0^{\lambda_1/c_1} t^{(c_2/c_1)k} e^{-t} dt\right]
\end{aligned} \tag{2.7}
$$

where $\Gamma(.)$ is a gamma function. If we write the identity given by Eq.(2.8) in the right hand side of the integral given by Eq.(2.7),

(2.8) $e^{-t} = \sum_{i=0}^{\infty} \frac{(-1)^i t^i}{i!}$

then Eq.(2.7) is rearranged as follows;

$$
\begin{aligned}
R &= 1 - \exp\left\{\lambda_1/c_1 + \lambda_2/c_2\right\} \\
&\quad \times \sum_{k=0}^{\infty} \frac{(-1)^k \left(\lambda_2/c_2\right)^k \left(c_1/\lambda_1\right)^{kc_2/c_1}}{k!} \\
&\quad\quad \times \left[\Gamma\left((c_2/c_1)k+1\right) - \sum_{i=0}^{\infty} \frac{(-1)^i(\lambda_1/c_1)^{(c_2/c_1)k+i+1}}{\left((c_2/c_1)k+i+1\right) i!}\right]
\end{aligned} \tag{2.9}
$$

If $c_1 = c_2 = c$, then R is in the form given as follows;

$$
\begin{aligned}
R = P\left(Y < X\right) &= \int_0^{\infty} P\left(Y < x\right) f_X\left(x\right) dx \\
&= \int_0^{\infty} \left[1 - \exp\left\{-\frac{\lambda_2}{c}(e^{cx}-1)\right\}\right] \lambda_1 e^{cx} \exp\left\{-\frac{\lambda_1}{c}(e^{cx}-1)\right\} dx \\
&= \frac{\lambda_2}{\lambda_1+\lambda_2}.
\end{aligned} \tag{2.10}
$$

3. Estimation of Stress-Strength Reliability

3.1. Maximum Likelihood Estimation

Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be the two independent random samples taken from the *Gompertz* distribution with parameters (c, λ_1) and (c, λ_2) respectively and let c be known. Then, likelihood and log-likelihood function based on the above samples are given as follows;

$$
\begin{aligned}
L\left(\boldsymbol{\theta}; \mathbf{x}, \mathbf{y}\right) &= \lambda_1^n \exp\left(c\sum_{i=1}^{n} x_i\right) \exp\left(-\frac{\lambda_1}{c}\sum_{i=1}^{n}(e^{cx_i}-1)\right) \lambda_2^m \\
&\quad \exp\left(c\sum_{i=1}^{m} y_i\right) \times \exp\left[-\frac{\lambda_2}{c}\sum_{i=1}^{m}(e^{cy_i}-1)\right]
\end{aligned} \tag{3.1}
$$

and

$$\ell\left(\boldsymbol{\theta};\mathbf{x},\mathbf{y}\right) = n\log\lambda_1 + c\sum_{i=1}^{n}x_i - \frac{\lambda_1}{c}\sum_{i=1}^{n}(e^{cx_i}-1)$$
(3.2)
$$+m\log\lambda_2 + c\sum_{i=1}^{m}y_i - \frac{\lambda_2}{c}\sum_{i=1}^{m}(e^{cy_i}-1)$$

respectively, where $\boldsymbol{\theta} = (\lambda_1, \lambda_2)$ is the parameter vector and subsequently the associated gradients are found as follows;

$$\begin{aligned}
\frac{\partial\ell\left(\boldsymbol{\theta};\mathbf{x},\mathbf{y}\right)}{\partial\lambda_1} &= \frac{n}{\lambda_1} - \frac{1}{c}\sum_{i=1}^{n}(e^{cx_i}-1) = 0,\\
\frac{\partial\ell\left(\boldsymbol{\theta};\mathbf{x},\mathbf{y}\right)}{\partial\lambda_2} &= \frac{m}{\lambda_2} - \frac{1}{c}\sum_{i=1}^{m}(e^{cy_i}-1) = 0.
\end{aligned}$$

Hence MLEs of the parameters λ_1 and λ_2 are obtained by

(3.3)
$$\hat{\lambda}_1 = \frac{n}{c^{-1}\sum_{i=1}^{n}\left(e^{cx_i}-1\right)}$$
(3.4)
$$\hat{\lambda}_2 = \frac{m}{c^{-1}\sum_{i=1}^{m}\left(e^{cy_i}-1\right)}$$

respectively. Using the invariance properties of the maximum likelihood estimation, $\widehat{R}_1$ that is the MLE of the R is obtained as follows;

$$(3.5)\quad \widehat{R}_1 = \frac{\widehat{\lambda}_2}{\widehat{\lambda}_1 + \widehat{\lambda}_2}.$$

Let $W = c^{-1}\sum_{i=1}^{n}\left(e^{cx_i}-1\right)$ and $V = c^{-1}\sum_{i=1}^{m}\left(e^{cy_i}-1\right)$. Then $\widehat{R}_1$ is calculated by

$$(3.6)\quad \widehat{R}_1 = \frac{\widehat{\lambda}_2}{\widehat{\lambda}_1 + \widehat{\lambda}_2} = \frac{m/V}{n/W + m/V}.$$

The following method can be used to find the distribution of $\widehat{R}_1$. Let $Z = m/V$ and we consider the below transformation

$$h:\begin{cases} r_1=\dfrac{m/v}{n/w+m/v} \\ z=m/v \end{cases} \text{ ; then } \begin{cases} v=m/z \\ w=\dfrac{nr_1}{z(1-r_1)} \end{cases}$$

and its jacobian is as follows;

$$J=\begin{vmatrix} \dfrac{nz\left(1-r_1\right)+nr_1z}{z^2\left(1-r_1\right)^2} & \dfrac{nr_1\left(1-r_1\right)}{z^2\left(1-r_1\right)^2} \\ 0 & -m/z^2 \end{vmatrix} = -\frac{nm}{z^3\left(1-r_1\right)^2}$$

so that $|J|=nm/\left\{z^3\left(1-r_1\right)^2\right\}$. Since $v,w>0$ implies $r_1,z>0$, we have

$$\begin{aligned} f_{\widehat{R}_1,Z} &= f_{W,V}\left(\frac{nr_1}{z(1-r_1)},m/z\right)|J| \\ &= f_W\left(\frac{nr_1}{z(1-r_1)}\right)f_V\left(m/z\right)|J| \\ &= \frac{\lambda_1^n\lambda_2^m m^m n^n r_1^{n-1}}{\Gamma\left(m\right)\Gamma\left(n\right)\left(1-r_1\right)^{n+1}}\frac{1}{z^{n+m+1}}\exp\left\{-\frac{nr_1\lambda_1}{z(1-r_1)}-\frac{m\lambda_2}{z}\right\}dz \end{aligned}$$

with $z>0$ and $0<r_1<1$. Then the distribution of $\widehat{R}_1$can be found as follows;

$$\begin{aligned} f_{\widehat{R}_1}\left(r_1\right) &= \frac{\lambda_1^n\lambda_2^m m^m n^n r_1^{n-1}}{\Gamma\left(m\right)\Gamma\left(n\right)\left(1-r_1\right)^{n+1}}\int_0^\infty\frac{1}{z^{n+m+1}}\exp\left\{-\frac{nr_1\lambda_1}{z(1-r_1)}-\frac{m\lambda_2}{z}\right\}dz \\ &= \frac{\Gamma\left(n+m\right)}{\Gamma\left(n\right)\Gamma\left(m\right)}\left(\frac{n\lambda_1}{m\lambda_2}\right)^n r_1^{n-1}\left(1-r_1\right)^{m-1} \\ &\quad\times\left\{1-r_1\left(1-\frac{n\lambda_1}{m\lambda_2}\right)\right\}^{-(n+m)} \end{aligned} \tag{3.7}$$

with $0<r_1<1$. For $s>0$, s^{th} moment of $\widehat{R}_1$ is given by

$$\begin{aligned} E\left(\widehat{R}_1^s\right) &= \int_0^\infty r_1^s f_{\widehat{R}_1}\left(r_1\right) \\ &= \frac{\Gamma\left(n+s\right)\Gamma(n+m)}{\Gamma\left(n+m+s\right)\Gamma\left(n\right)}\left(\frac{n\lambda_1}{m\lambda_2}\right)^n \\ &\quad\times F_{2,1}\left(\left(n+s,n+m\right),n+m+s,1-\frac{n\lambda_1}{m\lambda_2}\right), \end{aligned} \tag{3.8}$$

where $F_{p,q}(\mathbf{n},\mathbf{d},r)$ is the generalized hypergeometric function. This function is also known as Barnes's extended hypergeometric function. The definition of $F_{p,q}(\mathbf{n},\mathbf{d},r)$ is as follows;

$$(3.9)\quad F_{p,q}(\mathbf{n},\mathbf{d},r)=\sum_{k=0}^{\infty}\frac{r^k\prod_{i=1}^{p}\Gamma(n_i+k)\Gamma^{-1}(n_i)}{\Gamma(k+1)\prod_{i=1}^{q}\Gamma(d_i+k)\Gamma^{-1}(d_i)},$$

where $\mathbf{n}=[n_1,n_2,...,n_p]$, p is the number of operands of $\mathbf{n}$, $\mathbf{d}=[d_1,d_2,...,d_q]$ and q is the number of operands of $\mathbf{d}$. The above generalized hypergeometric function is quickly evaluated and readily available in standard software programmes such as Maple. For more details see Gradshteyn et al. (2000). By replacing $s=1$ in Eq.(3.8) the expected value of $\widehat{R}_1$ can be found as follows;

$$(3.10)\quad E(\widehat{R}_1)=\frac{n}{(n+m)}\left(\frac{n\lambda_1}{m\lambda_2}\right)^n F_{2,1}\left((n+1,n+m),n+m+1,1-\frac{n\lambda_1}{m\lambda_2}\right).$$

Fig. 1 shows the graphs of bias of the MLE as a function of the true reliability R for these cases: (a) n=5, m=3, (b) n=3, m=5, (c) n=10, m=10 and (d) n=15, m=15. MLE has relatively more bias for lower reliability values than higher ones when $m<n$ and MLE has relatively more bias for higher reliability values than lower ones when $m>n$. Also bias of the MLE tends to decrease in the case that the total sample size, $n+m$, increases.

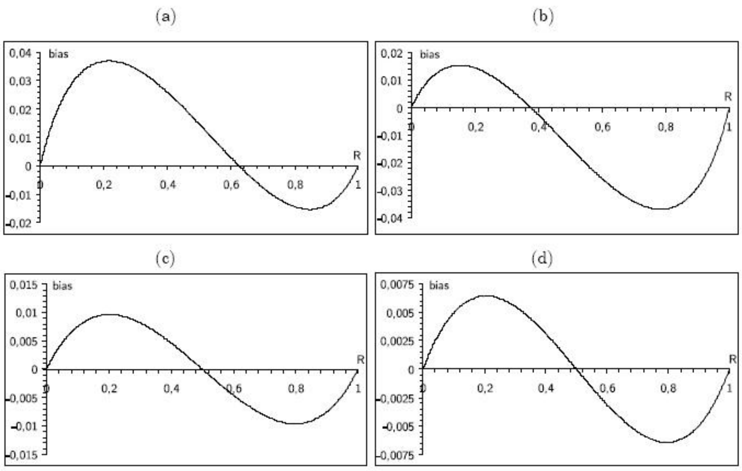

Fig. 1.The Bias curves of the MLE, for (a)$n=5$, $m=3$, (b)$n=3$, $m=5$, (c)$n=10$, $m=10$ and (d)$n=15$, $m=15$.

Using Eq. (3.8), the variance of the $\widehat{R}_1$ can be obtained as follows;

$$Var\left(\widehat{R}_1\right)$$

$$= E\left(\widehat{R}_1^2\right) - \left(E\left(\widehat{R}_1\right)\right)^2$$

$$= \frac{n\,(n+1)}{(n+m)\,(n+m+1)}\left(\frac{n\lambda_1}{m\lambda_2}\right)^n$$

$$\times F_{2,1}\left((n+2, n+m), n+m+2, 1-\frac{n\lambda_1}{m\lambda_2}\right)$$

$$-\left\{\frac{n}{(n+m)}\left(\frac{na_1}{ma_2}\right)^n\right.$$

(3.11)

$$\left.F_{2,1}\left((n+1, n+m), n+m+1, 1-\frac{n\lambda_1}{m\lambda_2}\right)\right\}^2$$

4. Confidence intervals

4.1. Exact confidence interval

Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be the two independent random samples taken from the *Gompertz* distribution with parameters (c, λ_1) and (c, λ_2) respectively and let c be known. Recall that $W = c^{-1}\sum_{i=1}^{n}\left(e^{cx_i}-1\right)$ and $V = c^{-1}\sum_{i=1}^{m}\left(e^{cy_i}-1\right)$ are independent gamma random variables with parameters (c, λ_1) and (c, λ_2) respectively. Also it can be easily shown that $2\lambda_1 W$ and $2\lambda_2 V$ are two independent chi-square random variables with $2n$ and $2m$ degrees of freedom respectively. Thus $\widehat{R}_1$ in Eq.(3.5) could be rewritten as $\left(1+\widehat{\lambda}_1/\widehat{\lambda}_2\right)^{-1}$. Using Eq.s(2.10), (3.3), (3.4) and (3.6) the MLE of $R_1$is obtained as follows;

(4.1) $$\widehat{R}_1 = \left(1+\frac{\lambda_1}{\lambda_2}\xi\right)^{-1},$$

where

(4.2) $$\xi = \frac{nRV}{m\,(1-R)\,W}$$

is an F distributed random variable with $(2m, 2n)$ degrees of freedom. ξ could be written as follows;

(4.3) $$\xi = \frac{R}{1-R}\left(\widehat{R}_1^{-1}-1\right).$$

Using ξ as a pivotal quantity, a $(1-\alpha)\,100\%$ exact confidence interval for R is obtained by

$$\left(\frac{F_{(\alpha/2)(2m,2n)}}{F_{(\alpha/2)(2m,2n)}+\widehat{R}_1^{-1}-1},\frac{F_{(1-\alpha/2)(2m,2n)}}{F_{(1-\alpha/2)(2m,2n)}+\widehat{R}_1^{-1}-1}\right), \tag{4.4}$$

where $F_{(a)(r,s)}$, is the a^{th} quantile of the F distribution with (r,s) degrees of freedom.
The other option is to find a $100(1-\alpha)\%$ lower confidence bound lcb for R. Then $(lcb,1)$ is a $100(1-\alpha)\%$ one-sided confidence interval for R. Hence for any $0<\alpha<1$, a $100\,(1-\alpha)\,\%$ lower confidence bound for R is

$$\frac{F_{(\alpha)(2m,2n)}}{F_{(\alpha)(2m,2n)}+\widehat{R}_1^{-1}-1}. \tag{4.5}$$

4.2. Asymptotic confidence interval

Let $X_1,X_2,\ldots,X_n$ and $Y_1,Y_2,\ldots,Y_m$ be the two independent random samples taken from the *Gompertz* distribution with parameters (c,λ_1) and (c,λ_2) respectively. The MLE $\widehat{R}_1$ in Eq.(3.5) is asymptotically normal with mean R and variance

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{\partial R}{\partial\lambda_1}\frac{\partial R}{\partial\lambda_2}I_{ij}^{-1} \tag{4.6}$$

where I_{ij}^{-1} is the $(i,j)th$ element of the inverse of the Fisher's information matrix which is given by

$$I=\begin{bmatrix}\frac{n}{\lambda_1^2} & 0\\ 0 & \frac{m}{\lambda_2^2}\end{bmatrix}$$

(Rao, 1965). Thus, the asymptotic variance of $\widehat{R}_1$ is as follows;

$$\frac{n+m}{nm}\widehat{R}_1^2\left(1-\widehat{R}_1\right)^2. \tag{4.7}$$

Hence an asymptotic $(1-\alpha)\,100\%$ confidence interval for R is obtained by

$$\left(\widehat{R}_1-z_{1-\alpha/2}\sqrt{\frac{n+m}{nm}}\widehat{R}_1\left(1-\widehat{R}_1\right),\widehat{R}_1+z_{1-\alpha/2}\sqrt{\frac{n+m}{nm}}\widehat{R}_1\left(1-\widehat{R}_1\right)\right), \tag{4.8}$$

where $z_{1-\alpha/2}$ is the $1-\alpha/2^{th}$ quantile of the standard normal distribution.

4.3. Simulation study

To study the performance of the confidence intervals, 50000 samples are simulated from the *Gompertz* distribution with the values of parameters $(\lambda_1, \lambda_2, c) = (1, 2, 1)$, $(1, 5, 1)$, $(5, 5, 1)$ and different sample size of n and m. It is important to examine how well our proposed methods work for constructing confidence intervals. In this section, the approximate confidence intervals based on asymptotic properties of the MLEs are compared with the exact confidence intervals in terms of coverage probabilities. The simulation results are shown in Table 1 and Table 2. The coverage probabilities of the exact confidence intervals for R are all close to the desired level of 0.95, but the coverage probabilities of the approximate confidence intervals for R are not so close to 0.95. The coverage probabilities of the approximate confidence intervals are close to 0.95, virtually for $n \geq 50$ and $m \geq 50$.

		Case 1: $(1,2,1)$		Case 2: $(1,5,1)$		Case 3: $(5,5,1)$	
n	m	Exact	Asymp.	Exact	Asymp.	Exact	Asymp.
10	5	0.95028	0.89274	0.94972	0.89098	0.95036	0.89884
10	10	0.94942	0.91642	0.95060	0.92038	0.95134	0.91736
10	15	0.95064	0.92488	0.94882	0.92842	0.95030	0.92128
10	20	0.94740	0.92574	0.94984	0.93468	0.95162	0.92472
10	25	0.94812	0.92772	0.95028	0.93570	0.94948	0.92248
10	30	0.95092	0.93180	0.95196	0.93904	0.95076	0.92546
10	35	0.95000	0.93112	0.94974	0.93790	0.94878	0.92294
10	40	0.95030	0.93132	0.94962	0.93916	0.94896	0.92426
10	45	0.94918	0.93160	0.94944	0.94050	0.94912	0.92444
10	50	0.95082	0.93306	0.95150	0.94162	0.94920	0.92512

Table 1. Coverage probabilities for the proposed methods and the MLEs of R for various values of $(\lambda_1, \lambda_2, c)$ (n fixed, m increased)

		Case 1: $(1,2,1)$		Case 2: $(1,5,1)$		Case 3: $(5,5,1)$	
m	n	Exact	Asymp.	Exact	Asymp.	Exact	Asymp.
10	5	0.95014	0.90598	0.94932	0.91628	0.95000	0.89920
10	10	0.95090	0.91964	0.95196	0.92010	0.94916	0.91422
10	15	0.95084	0.91892	0.94974	0.91820	0.94844	0.91862
10	20	0.94880	0.92086	0.95156	0.91812	0.95046	0.92402
10	25	0.95144	0.92292	0.95168	0.91994	0.95092	0.92472
10	30	0.94946	0.92092	0.94974	0.91400	0.95070	0.92488
10	35	0.94926	0.91960	0.94886	0.91590	0.94878	0.92520
10	40	0.95248	0.92338	0.94914	0.91508	0.94964	0.92586
10	45	0.94990	0.92042	0.94926	0.91434	0.95174	0.92594
10	50	0.95146	0.92112	0.94850	0.91374	0.95106	0.92662

Table 2. Coverage probabilities for the proposed methods and the MLEs of R for various values of $(\lambda_1, \lambda_2, c)$ (m fixed, n increased)

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