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Mathematical Model of the Impact of Pressure Drop on Human Body

Ahmad Reza Haghighi

Department of Mathematics, Urmia University of Technology,Urmia, Iran
e-mail: ah.haghighi@gmail.com

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Abstract. Mathematical model for the impact of pressure drop on the human body has been investigated in the present studies. The studies has been aimed at personnel (army and mountaineer) who would be prone for higher altitude effect on the body and to suggest them appropriate measures (as a precautionary or advisory purpose) who either will be getting inducted onto higher altitudes venturing onto higher peaks. The model accounts for heights of altitudes ranging from 4000-6000 meters and accounting for all the possible cardiovascular diseases.

Key words: Physiological fluid dynamics; Heart Attack; Effect of Low pressure; Higher altitude.
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1. Introduction

In the present studies physiological aspect of the blood flow has been modeled with a view to estimate the physiological flow parameters such as pressure, against adverse conditions. The study is basically aims at as an advisory and precautionary mode. Efforts have been made to compare the results with practical situations available in the literature with respect to myocardial infarction [heart attack]. Blood flow modeling has paved the way for understanding the intricacy of the fluid flow pattern in the human body[1,2].The importance of blood flow in Cardiovascular system has been highlighted by Young[3]. Later the models have been refined by accounting it for pulsatile aspect [4] and the effects of blood cells [5-6] by using micro-continuum theories [7-9].Effects of body acceleration and magnetism have also been studied on the blood flows [10-12]. In the present model, blood is assumed to be represented by a couple stress fluids [7] and the model has been developed for the straight tube [Figure1].

2. Analysis

It is assumed that the flow is steady and laminar and turbulence effects in the body are neglected

$$\eta\nabla^4 u - \mu\nabla^2 u + \sigma B_0^2 u = -\frac{\partial p}{\partial z} + \rho G \tag{1}$$

Where $u(r)$ is the velocity in the axial direction, ρ and μ are the density and viscosity of blood, η is the couple stress parameter, σ is the electrical conductivity, B_0 is the external magnetic field and r is the radial coordinate.

$$\nabla^2 = \frac{1}{r}\frac{1}{\partial r}(r\frac{\partial}{\partial r}) \tag{2}$$

The flow geometry of blood flow has been shown in figure 1.

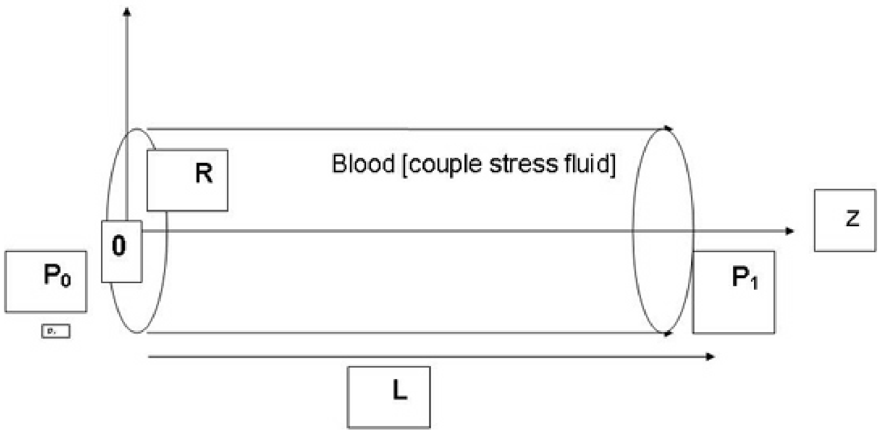

Figure 1. Blood flow in straight tube

The Pressure gradient and body acceleration are assumed to be of the form [10]

$$-\frac{\partial p}{\partial z} = A_0 + A_1 \quad , \quad G = a_0\cos(\phi) \tag{3}$$

Where A_0 the steady-state part of pressure gradient, A_1 is the amplitude of the oscillatory part, a_0 is the amplitude of body acceleration,ϕ the phase difference,z is the axial distance. Flow variables have been normalized by using flowing relations:

$$u^* = \frac{u}{\omega R}, r^* = \frac{r}{R}, A_0^* = \frac{R}{\mu\omega}A_0, A_1^* = \frac{R}{\mu\omega}A_1, a_0^* = \frac{\rho R}{\mu\omega}a_0, Z^{ast} = \frac{z}{R}. \tag{4}$$

Where $\omega = 2\pi f$ and f is heart pulse frequency. Equation (1) simplifies to

$$\nabla^4 u - \bar{a}^2\nabla^2 u = \bar{a}^2 A_0 + \bar{a}^2 A_1 + \bar{a}^2 a_0\cos(\phi) - \bar{a}^2 H^2 u \tag{5}$$

Where $\bar{a}^2 = \bar{a}^{*^2}\frac{\mu}{\mu_1}$, couple stress parameter,$H = H^*\sqrt{\frac{\mu}{\mu_1}}$ is the Hartmann number ,and R is the radius pipe.

$$\bar{a}^{*^2} = \frac{R^2\mu_1}{\eta}, H^* = B_0 R\sqrt{\frac{\sigma}{\mu_1}} \tag{6}$$

Shakera and Rathod [10] have analyzed the model for velocity computation for the straight tube and this expression for velocity has been taken for further investigations in the present studies with appropriate changes for the present analysis if straight tube .

$$u(r^*, t) = \Sigma_{n=1}^{\infty}\frac{J_0(r^*\lambda_n)\bar{a}^2}{\lambda_n J_(\lambda_n)}\frac{[A_0 + A_1 + a_0\cos\phi]}{[\lambda_n^4 + \bar{a}^2(\lambda_n^2 + H^2)]} \tag{7}$$

And J_0 and J_1 are the Bessel functions of order zero and one respectively and λ_n are the boots of equation $J_0(r) = 0$. Uses of transform techniques [13] have been made use in solving equation(5).

3. Flow Rate

$$Q = 2\pi\int_0^1 ru(r)dr \quad , \quad Q = \Sigma_{n=1}^{\infty}\frac{\bar{a}^2}{\lambda_n^4}\frac{[A_0 + A_1 + a_0\cos\phi]}{[\lambda_n^4 + \bar{a}^2(\lambda_n^2 + H^2)]} \tag{8}$$

4. Resistance to Flow

Resistance to flow λ has been computed by using following relation

$$\lambda = \frac{p_1 - p_0}{Q} \qquad \lambda^* = \lambda(\frac{p'}{\lambda_1\omega}) \tag{9}$$

Wherep' is the pressure used for normalizing p.

5. Computation of Pressure with altitude

The equation which computes atmospheric pressure with altitude (as per International standard) is given by[14]

$$p_0(z) = 1013 \times \lfloor(1 - 220558 \times 10^{-6} \times z)^{525611}\rfloor \tag{10}$$

Here 1013.25 is the pressure at sea level in mil/bar, z is the altitude in meters. Knowing $p_0(z)$ and the relation for resistance to flow, the pressure variation on the human body have been computed by the following method. We have

$$\lambda_i = \frac{p_i - p_0}{Q}\ ,\ p_i = p_0 + Q\lambda_i. \tag{11}$$

The units for p_i pressure [equation (12)] are dynes/cm^2. In order make $Q\lambda_i$ has same units that of p_i,λ_i and Q have been converted to the following expression

$$\bar{\lambda} = \lambda_i \lambda_0 \ , \ \bar{Q} = QQ_0 \qquad (12)$$

Here λ_0 and Q_0 are taken in the form [5]

$$\lambda_o = \frac{\mu_1 L}{\pi R_0^4} \ \text{ and } \ Q_0 = \frac{\pi L R_0^2}{t} \qquad (13)$$

The values of μ_1 [viscosity of plasma] is taken to be 1.2×10^{-2} Poise, $L = 4cm$, $R = 1cm$. The value of $\lambda_0 = 0.01527 dynes/cm^3$ and $Q_0 = 12.571 cm^3/sec$ has been obtained and incorporated in the computation of p_i. It is assumed that, the atmospheric pressure variation has been accounted in the initial development of the model.

6. Results and Discussion

For the computation of resistance to flow λ_i for different diseases, data for μ and μ_1 are required and this data has been taken from [5] and shown in Table 1.The values of p (for p_i at r^*) Normal blood and for different diseases has been computed and tabulated in Table [2-3]. Also selected variations of pi with various flow parameters have been shown in figure [2]. Results indicate that, values for pressure at higher altitudes, in the blood diseases; Polycythemia, Pleasma cell Dyscrasis values are higher to Normal blood.[Table 2-3]. The analysis confirms that, pressure decreases with the increases in the altitude. Also it is observed that, pressure values decreases with the increases in H^* [magnetic effects] (Figure 2).

Table 1. Computed pressure for various higher altitudes.

Numbers	Deceases	μ, cP	μ_1, cP
1	Normal Blood	3.81	1.2
2	Polycythemia	6.75	1.2
3	Plasma cell Dyscrasis	4.99	1.2
4	Hb SS	3.29	1.2

Table 2. computed pressure for various higher altitudes.
$\bar{a}^* = 4$,$a_0 = 3$,$H^* = 2$,$A_0 = 2$,$A_1 = 4$,$\frac{L}{R} = 4$,$\phi = \pi/12$,$r = 1$

z m	$p_0(z)$ mb	Normal Blood p_i, mb	Polycythemia p_i, mb	Plasam cell p_i, mb	Hb,SS p_i, mb
4000	616.38	623.09419	626.00975	624.37323	622.46152
4500	577.26	583.97399	586.88955	585.25303	583.34132
5000	540.17	546.88899	549.80455	548.16803	546.25632
5500	505.04	511.75709	514.67265	513.03613	511.12442
6000	471.78	478.49879	481.41435	479.77783	477.86612

Diseases	Normal Blood	Polycythemia	Plasma cell	Hb SS
Flow Rate Q	1.079490	0.7452251	0.9147911	1.171579

Table 3. computed pressure for various higher altitudes.
$\bar{a}^* = 4, a_0 = 3, H^* = 4, A_0 = 2, A_1 = 4, \frac{L}{R} = 4, \phi = \pi/12, r = 1$

z m	$p_0(z)$ mb	Normal Blood p_i, mb	Polycythemia p_i, mb	Plasam cell p_i, mb	Hb,SS p_i, mb
4000	616.38	619.20184	619.96230	620.62066	619.45205
4500	577.26	580.08164	580.84210	581.50046	580.33185
5000	540.17	542.99664	543.75710	544.41546	543.24685
5500	505.04	511.75709	514.67265	513.03613	511.12442
6000	471.78	474.60644	475.36690	476.02526	474.85665

Diseases	Normal Blood	Polycythemia	Plasma cell	Hb SS
Flow Rate Q	0.380062	0.2384854	0.3076084	0.4298614

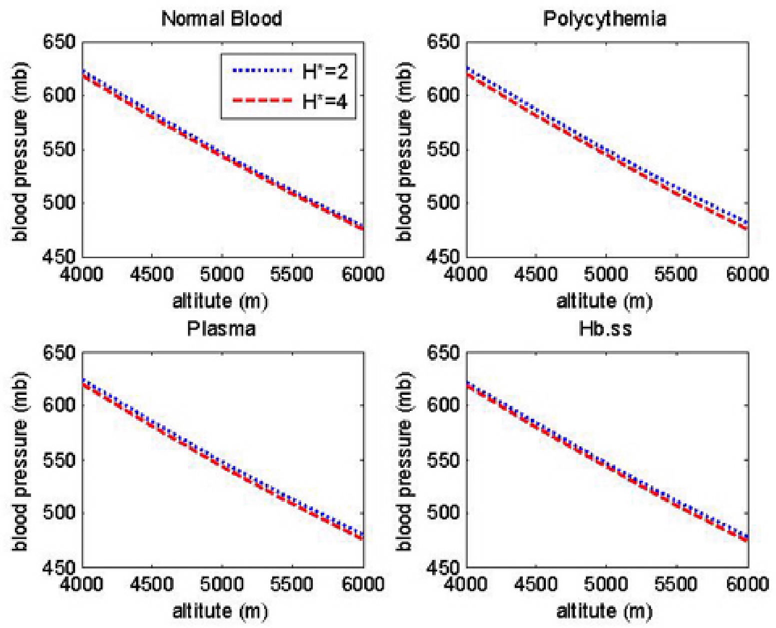

Figure 2. $\bar{a}^* = 4$, $a_0 = 3$, $A_0 = 2$, $A_1 = 4$, $\frac{L}{R} = 4$, $\phi = \pi/12$, $r = 1$

7. Conclusions

Mathematical model for blood flow has been developed in the present model with a view to correlate the finding with its impact on higher altitude [for the benefit of mountaineer or service, personnel getting inducted onto higher altitudes].This has been done with a view to analyze its enhanced effect on the higher altitude for those who will be or already prone for Cardiovascular problems [disease related to myocardial infraction].The diseases chosen for the present computation propose are Polycythemia,Pleasma cell dyscrasis and for Hb SS(sickle cell Anemia).The data has been collected from earlier published results [6, 11]. Resistance to flow values has been made use of for computing the pressure computations at higher altitudes.The computed pressures for different altitudes have been shown in Table 2 and 3.The results indicate that, pressure drops with altitude and its effects enhances(lowering of pressure values) with the diseased case. lowering value of pressure in diseases case signifies that the person prone for heart disease can perform to some extent better in lower altitudes but certainly not advisable in higher altitudes. Such measures are useful for the doctors and the service personal for inducting the staff on the higher altitudes. The findings of the present model are required to be tested for its actual utility point of view.Once the results are confirmed the model can be used for other related diseases.

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