Selçuk J. Appl. Math.
Vol. 11. No.2. pp. 109-122, 2010

Selçuk Journal of Applied Mathematics

Numerical Study of Heat and Mass Transfer in Magneto Hydrodynamic Flow past a Vertical Plate with Constant Injection and Heat Flux

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Received Date: April 7, 2010
Accepted Date: August 15, 2010

Abstract. We have studied heat and mass transfer in an unsteady MHD flow of an incompressible, electrically conducting, and viscous fluid. It is considered that the influence of the uniform magnetic field normal to the flow. Numerical results for temperature, velocity, concentration, have been obtained and shown graphically for suitable parameters like Grashoff number, mass Grashoff number, Prandtl number and Schmidt number. Rate of heat transfer and mass transfer are studied. The results obtained are discussed with the help of graphs and tables to observe effect of various parameters concerned in the problem under investigation. The main conclusions of this study have been given.

Key words: Injection; Heat Flux; Heat and Mass transfer; Numerical solution.
2000 Mathematics Subject Classification. 80A20.

1. Introduction

Investigation of magneto-hydrodynamic (MHD) flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. This study has been largely concerned with the flow and heat and mass transfer characteristics in various physical situations. Vajravelu and Hadjinicolaou (1997) studied convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Anjalidevi and Kandasamy (1999) investigated effects of chemical reaction, heat and mass transfer on laminar flow along a semi infinite horizontal plate. Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and

constant heat flux was studied by Acharya *et al.* (2000). Sahoo *et al.* (2003) have studied MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity was investigated by Singh *et al.* (2003). Chamkha (2004) investigated unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Hazem (2006) studied on the effectiveness of uniform suction and injection on unsteady rotating disk flow in porous medium with heat transfer. Chaudhary and Jha (2008) have studied heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with Hall effect. Mahdy *et al.* (2009) have investigated heat and mass transfer in MHD free convection along a vertical wavy plate with variable surface heat and mass flux. Samad and Mohebujjaman (2009) have studied MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation.
Because of enormous practical applications of injection to problems of boundary layer control, thermal protection of high energy flows and recently to those of seeding processes to enhance possible MHD effects and due to intricacy analysis to investigate combined effects of heat and mass transfer with injection of an MHD flow, we have been fascinated to and motivated towards this direction. Our main purpose is to investigate numerically the problem of combined heat and mass transfer of an unsteady MHD flow past an infinite plate with injection. The results of this study are discussed for various numerical values of the parameters which suits for the case of injection.

2. Mathematical Formulation

Here an unsteady two dimensional free convective flow of an electrically conducting viscous and incompressible fluid past an infinite, porous and vertical plate with constant injection and heat flux is considered. A magnetic field B_0 is applied perpendicular to the plate. A system of rectangular coordinate axes $o\,x_1y_1z_1$ is taken such that $y_1 = 0$ on the plate and z_1 is along its leading edge. All the fluid properties are considered. The influence of the density variation with temperature is considered only in the body force term. Its influence in other terms of the momentum and the energy equations is assumed to be negligible. The variation of expansion coefficient with temperature is considered to be negligible. This is the well-known Boussinesq approximation. Thus, under these assumptions, the physical variables are functions of y_1 and t_1 only and the problem is governed by the following system of equations

(1) continuity equation : $$\frac{\partial v_1}{\partial y_1} = 0,$$

(2) momentum equations : $$\frac{\partial u_1}{\partial t_1} + v_1\frac{\partial u_1}{\partial y_1} = g\beta(T_1 - T_\infty) + \frac{\nu\partial^2 u_1}{\partial y_1^2} - \frac{\sigma B_0^2 u_1}{\rho},$$

(3) energy equation : $\frac{\partial T_1}{\partial t_1} + v_1 \frac{\partial T_1}{\partial y_1} = \frac{k\partial^2 T_1}{\partial y_1^2}$,

(4) mass transfer equations : $\frac{\partial C_1}{\partial t_1} + v_1 \frac{\partial C_1}{\partial y_1} = \frac{D\prime\partial^2 C_1}{\partial y_1^2}$.

The initial and boundary conditions of the problem are

$$t_1 \leq 0,$$
$$u_1(y_1, t_1) = 0,$$
$$T_1(y_1, t_1) = T_\infty,$$
$$C_1(y_1, t_1) = C_\infty; \tag{5}$$

$$t_1 > 0,$$
$$u_1(0, t_1) = V_0,$$
$$T_1(0, t_1) = T_P + \varepsilon(T_p - T_\infty)e^{i\omega_1 t_1},$$
$$C_1(0, t_1) = C_\infty + \varepsilon(C_p - C_\infty)At_1, \quad \text{at} \quad y_1 = 0; \tag{6}$$

$$t_1 > 0,$$
$$u_1(\infty, t_1) \to 0,$$
$$T_1(\infty, t_1) \to T_\infty,$$
$$C_1(\infty, t_1) \to C_\infty, \quad \text{as} \quad y_1 \to \infty. \tag{7}$$

Since the plate is assumed to be porous type and through it suction with uniform velocity occurs, equation (1) integrates to

$$v_1 = -v_0$$

which is the constant suction velocity. Here, u is the velocity of the fluid, T_1 is the temperature of the fluid, T is the dimensionless temperature, T_p is the temperature of the fluid near the plate, T_∞ is the temperature of the fluid far away from the plate, C_1 is the concentration of the species, C_p is the concentration near the plate, C_∞ is the concentration far away from the plate, C is dimensionless concentration, g is the acceleration due to gravity, β is the coefficient of volume expansion for heat transfer, β' is the coefficient of volume expansion for concentration, ν is the kinematic viscosity, σ is the scalar electrical conductivity, ω is the frequency of oscillation, ε is a constant, B_0 is the applied uniform magnetic field, ρ is the density of the fluid, k is the thermal conductivity, r is the injection parameter, D' is the molecular diffusivity, and t is the time.
From equation (1) we observe that v_1 is independent of space co-ordinates and may be taken as constant. We define the following non-dimensional variables

and parameters.

$$t = \frac{t_1 V_0^2}{4v}, \qquad y = \frac{V_0 y_1}{4v},$$

$$u = \frac{u_1}{V_0}, \qquad T = \frac{T_1 - T_\infty}{T_p - T_\infty}, \qquad C = \frac{C_1 - C_\infty}{C_p - C_\infty},$$

$$P_r = \frac{\nu}{k}, \qquad S_c = \frac{\nu}{D'}, \tag{8}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \qquad G_r = \frac{\nu g \beta (T_p - T_\infty)}{V_0^3}, \quad r = \frac{v_1}{V_0},$$

$$G_m = \frac{\nu g \beta' (C_P - C_\infty)}{V_0^3}, \quad \omega = \frac{r v \omega_1}{V_0^2}.$$

Now taking into account equations (5), (6), (7) and (8), equations (2), (3) and (4) reduce to the following non-dimensional form

$$\frac{\partial u}{\partial t} - r\frac{\partial u}{\partial y} = r\frac{\partial^2 u}{\partial y^2} + rG_r T - rMu + rG_m C, \tag{9}$$

$$\frac{\partial T}{\partial t} - r\frac{\partial T}{\partial y} = \frac{r}{P_r}\frac{\partial^2 T}{\partial y^2}, \tag{10}$$

$$\frac{\partial C}{\partial t} - r\frac{\partial C}{\partial y} = \frac{r}{S_c}\frac{\partial^2 C}{\partial y^2}. \tag{11}$$

with

$$\begin{aligned} &t \le 0,\\ &u(y,t) = 0,\\ &T(y,t) = 0,\\ &C(y,t) = 0; \end{aligned} \tag{12}$$

$$\begin{aligned} &t > 0,\\ &u(0,t) = 0,\\ &T(0,t) = 1 + \varepsilon e^{i\omega t},\\ &C(0,t) = t, \quad \text{at } y = 0; \end{aligned} \tag{13}$$

$$\begin{aligned} &t > 0,\\ &u(\infty,t) = 0,\\ &T(\infty,t) = 0,\\ &C(\infty,t) = 0, \quad \text{as } y \to \infty. \end{aligned} \tag{14}$$

The Grash of number $G_r > 0$ represents external cooling of the plate and $G_r < 0$ denotes external heating of the plate. G_m the modified Grshof number, S_c the Schmidt number, P_r the Prandtl number and r is the injection parameter.

3. Method of solution

In order to solve these unsteady, linear coupled equations (9) to (11) under the conditions (12) to (14), an implicit method of Crank-Nicolson type has been employed. To obtain the finite difference equations, the region of the flow is divided into a grid of mesh points (T_i, Y_j). The values of the dependent variables T, u and C at the nodal points along the plane $y = 0$ are given by $T(0, t)$, $u(0, t)$ and $C(0, t)$ hence are known from the boundary conditions. Let Δk and Δh represents the uniform step lengths in x and y directions. We need a scheme to find single values at next time level in terms of known values at an earlier time level. A forward difference approximation for the first order partial derivatives of u, T and C with respect to t and y and a central difference approximation for the second order partial derivative of u, T and C with respect to y are used. On introducing finite difference approximations for $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial T}{\partial t}, \frac{\partial T}{\partial y}, \frac{\partial^2 T}{\partial y^2}, \frac{\partial C}{\partial t}, \frac{\partial C}{\partial y}, \frac{\partial^2 C}{\partial y^2}$ as

$$\left(\frac{\partial T}{\partial t}\right)_{i,j} = \frac{T_{i,j+1} - T_{i,j}}{(\Delta k)}; \quad \left(\frac{\partial T}{\partial y}\right)_{i,j} = \frac{T_{i+1,j+1} - T_{i-1,j+1} + T_{i+1,j} - T_{i-1,j}}{4(\Delta h)};$$

$$\left(\frac{\partial C}{\partial t}\right)_{i,j} = \frac{C_{i,j+1} - C_{i,j}}{(\Delta k)}; \quad \left(\frac{\partial C}{\partial y}\right)_{i,j} = \frac{C_{i+1,j+1} - C_{i-1,j+1} + C_{i+1,j} - C_{i-1,j}}{4(\Delta h)};$$

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{(\Delta k)}; \quad \left(\frac{\partial u}{\partial y}\right)_{i,j} = \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta h)};$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j} + T_{i+1,j+1} - 2T_{i,j+1} + T_{i-1,j+1}}{2(\Delta h)^2};$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2(\Delta h)^2};$$

$$\left(\frac{\partial^2 C}{\partial y^2}\right)_{i,j} = \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j} + C_{i+1,j+1} - 2C_{i,j+1} + C_{i-1,j+1}}{2(\Delta h)^2}, \tag{15}$$

The finite difference approximation of equations (9) to (11) are obtained on substituting equation (15) into equations (9)-(11)

$$2u_{i,j+1} - \left(\frac{1}{2} + \frac{\Delta k}{\Delta h}\right) u_{i+1,j+1} + \left(\frac{\Delta k}{\Delta h} - \frac{1}{2}\right) u_{i-1,j+1}$$
$$= \left(\tfrac{1}{2} + \tfrac{\Delta k}{\Delta h}\right) u_{i+1,j} + \left(-\tfrac{\Delta t}{\Delta y} + \tfrac{1}{2}\right) u_{i-1,j} + rG_r \Delta t T_{i,j} + rG_m \Delta t C_{i,j} - rM \Delta t u_{i,j}, \tag{16}$$

$$\left(1 + \frac{r}{P_r}\right) T_{i,j+1} + \left(\frac{\Delta t}{\Delta y} - \frac{2}{P_y}\right) T_{i-1,j+1} - \left(\frac{\Delta t}{\Delta y} + \frac{2}{P_r}\right) T_{i+1,j+1}$$
$$= \left(\frac{\Delta t}{\Delta y} + \frac{2}{P_y}\right) T_{i+1,j} + \left(\frac{2}{P_y} - \frac{\Delta t}{\Delta y}\right) T_{i-1,j} + \left(1 - \frac{r}{P_r}\right) T_{i,j}, \tag{17}$$

$$\left(1+\frac{r}{S_C}\right)C_{i,j+1}+\left(\frac{\Delta k}{\Delta h}-\frac{1}{2S_C}\right)C_{i-1,j+1}-\left(\frac{\Delta k}{\Delta h}+\frac{1}{S_c}\right)C_{i+1,j+1}$$

$$=\left(\frac{2}{S_c}+\frac{\Delta t}{\Delta y}\right)C_{i+1,j}+\left(\frac{2}{S_c}-\frac{\Delta t}{\Delta y}\right)C_{i-1,j}+\left(1-\frac{r}{S_C}\right)C_{i,j}\,. \tag{18}$$

4. Numerical Computations

To get the numerical solutions of the temperature T, velocity u and concentration C, we have taken the aid of the computer by developing a code in Mathematica5.0. The logic of the program is divided three steps as follows:

Step 1. main, initially it creates three tables to hold the numerical solutions of temperature, velocity and concentration whose coefficients are allotted in the step 2. After this, it calculates the numerical values at the next time level. In order to do this, it uses another sub- module namely Tri-diagonal, which solves the tri-diagonal matrix by using successive over relaxation method with complete pivoting. Further it moves to the step3, for listing the numerical solutions.

Step 2. Coeff Mat, we know that all the terms and their coefficients on right hand side of equations (16), (17) and (18) are known values from initial and boundary conditions. At every time step, for different values of 'i', the finite difference approximation of equation (18) gives a linear system of equations. Then, for $j = 0$ and $i = 1, 2, \dots n - 1$, equation (18) gives a linear system of $(n - 1)$ equations for the $(n - 1)$ unknown values of 'C' in the first time row in terms of known initial and boundary values. This module maintains coefficients of this linear system of equations. Similarly the above process repeats for the remaining equations (17) and (18) to obtain the numerical values of T and C.

Step 3. Tabulation, It lists the numerical solution at every time step level. By making use of T and C into equation (16), the numerical solutions for 'u' are obtained.

Code for numerical solutions of temperature profiles for $P_r = 0.733$ for the case of injection

```
CNgrid[n_, m_] :=
    Module[{i, j},
        u = Table[1, {n}, {m}];
        For[i = 1, i ≤ n, i + +, u[[i, 1]] = f[i];];
        For[j = 1, j ≤ m, j + +, u[[1, j]] = g1[j]; u[[n, , j]] = g2[j]; ]; ];

TriDiagonal[a0_, d0_, c0_, b0_] :=
    Module[{a = a0, b = b0, c = c0, d = d0, k, m, n = Length[b0], x},
        For[k = 2, k ≤ n, k + +,
            d[[k]] = d[[k]] − (a[[k−1]]/d[[k−1]]) ∗ c[[k−1]];
```

$b_{[[k]]} = b_{[[k]]} - (a_{[[k-1]]}/d_{[[k-1]]}) * b_{[[k-1]]};];$

$x = \text{Table}[0, \{n\}];\ x_{[[n]]} = b_{[[n]]}/d_{[[n]]};$

$\text{For } [k = n-1; 1 \leq k; k--, X_{[[k]]} = (b_{[[k]]} - c_{[[k]]} * x_{[[k+1]]})/d_{[[k]]};];$

Return $[x]$;];

Tabulation :

Module[{},

Print["Complete Table"];

Print[" t y Numerical Solution"];

Print["================================"];

result=Table["————", $\{(m * n) + m - 20\}, \{5\}$];

Input data:

$a = 1.0;\ b = 0.1;\ c = 1;\ n = 21;\ m = 41;\ r = -4;$

$F[x\_] = 0;\ G_1[t\_] = 1.0;\ G_2[t\_] = 0.0;$

$h = a/(n-1);\ k = b/(m-1);$

$f[i\_] = F[h(i-1)];\ g_1[j\_] = G_1[k(j-1)];\ g_2[j\_] = G_2[k(j-1)];$

CNgrid$[n, m]$; $r = (c^2 * k)/h^2;$

$Va = Vc = \text{Table}[-1, \{n-1\}]; Va_{[[n-1]]} = Vc_{[[1]]} = 0; Vd = \text{Table}[2+(2/r), \{n\}];$

$Vd_{[[1]]} = Vd_{[[n]]} = 1;$

$b = \text{Table}[0, \{n\}];$

$\text{For}[j = 2, j \leq m,\ j++,\ b_{[[1]]} = g_1[j];\ b_{[[n]]} = g_2[j];$

$$\begin{aligned}\text{For } [i = 2, i \leq n-1, i++,\\ b_{[[i]]} = (0.5 - ((R*k)/(4*h))) * u_{[[i-1,j-1]]} + u_{[[i,,j-1]]}\\ +(0.5 + ((R*k)/(4*h)))) * u_{[[i+1,j-1]]}\\ +((R*k)/(4*h)) * (u_{[[i+1,j]]} - u_{[i-1,j]]});];\end{aligned}$$

$u_{[[All,j]]}$ =TriDiagonal $[Va, Vd, Vc, b]$;

Print[NumberForm[TableForm[N[Transpose[Chop[u]]],TableSpacing − >{0,2}]]];

Print[TableForm[result,TableSpacing − > {0, 2}]];

Output:

Table 1. Numerical solutions of Temperature profiles for $P_r = 0.733$ for the case of injection

t	y	Numerical solution
0.1	0	1
0.1	0.05	0.923611
0.1	0.1	0.847906
0.1	0.15	0.773549
0.1	0.2	0.701163
0.1	0.25	0.631316
0.1	0.3	0.564501
0.1	0.35	0.501127
0.1	0.4	0.441507
0.1	0.45	0.385855
0.1	0.5	0.334278
0.1	0.55	0.286782
0.1	0.6	0.24327
0.1	0.65	0.203553
0.1	0.7	0.167353
0.1	0.75	0.134315
0.1	0.8	0.104015
0.1	0.85	0.075975
0.1	0.9	0.049669
0.1	0.95	0.024538
0.1	1	0

Code for numerical solutions of velocity profiles for $P_r = 0.733$

```
CNgrid[n_, m_]:=
    Module[{i, j},
        u = Table[1, {n}, {m}]; su = Table[1, {n}, {m}];
        For[i = 1, i ≤ n, i + +, u[[i,1]] = f[i]; su[[i,1]] = f[i]; ];
        For[j = 1, j ≤ m, j + +, u[[1,j]] = g1[j]; su[[1,j]] = g1[j];
            u[[n,j]] = g2[j]; su[[n,j]] = g2[j]; ];

TriDiagonal[a0_, d0_, c0_, b0_] :=
    Module[{a = a0, b = b0, c = c0, d = d0, k, m, n = Length[b0], x},
        For[k = 2, k ≤ n, k + +, d[[k]] = d[[k]] − (a[[k−1]]/d[[k−1]]) ∗ c[[k−1]];
            b[[k]] = b[[k]] − (a[[k−1]]/d[[k−1]]) ∗ b[[k−1]];
        x = Table[0, {n}]; x[[n]] = b[[n]]/d[[n]];
        For[k = n − 1, 1 ≤ k, k − −, x[[k]] = (b[[k]] − (c[[k]] ∗ x[[k+1]]))/d[[k]]; ];
        Return[x]; ];
            Module[{},
                Print["Complete Table"];
```

Print[" t y Numerical Solution"];
Print["==========================="];
result=Table["———", $\{(m*n)+m-20\},\{5\}$];

Input data:
$a=1.0; b=0.1; c=1; sc=1; n=21; m=41; G_r=2; M=3; r=-0.5;$
$F[x\_]=0; G_1[t\_]=1.0; G_2[t\_]=0.0;$
$h=a/(n-1); k=b/(m-1);$
$f[i\_]=F[h(i-1)]; g_1[j\_]=G_1[k(j-1)]; g_2[j\_]=G_2[k(j-1)];$ CNgrid$[n,m]$;
$r=(c^2*k)/h^2; sr=(sc^2*k)/h^2; tr=0.5;$

$Va=Vc=\text{Table}[-1,\{n-1\}]; Va_{[[n-1]]}=Vc_{[[1]]}=0; Vd=\text{Table}[2+(2/r),\{n\}];$
$sVd=\text{Table}[2+(2/sr),\{n\}]; Vd_{[[1]]}=Vd_{[[n]]}=1; sVd_{[[1]]}=sVd_{[[n]]}=1;$

$b=\text{Table}[0,\{n\}];$
$\text{For}[j=2, j\leq m, j++, b_{[[1]]}=g_1[j]; b_{[[n]]}=g_2[j];$

$\text{For }[i=2, i\leq n-1, i++, b_{[[i]]}=u_{[[i-1,j-1]}+((2/sr)-2)*u_{[[i,j-1]}+u_{[[i+1,j-1]};];$
$u_{[[All,j]]}=\text{TriDiagonal}[V_a,V_d,V_c,b];];$

$sb=\text{Table}[0,\{n\}]$
$\text{For}[j=2, j\leq m, j++, sb_{[[1]]}=g_1[j]; sb_{[[n]]}=g_2[j];$
$\text{For}[i=2, i\leq n-1, i++,$
$Sb_{[[i]]}=su_{[[i-1,j-1]]}+((2/sr)-2)*su_{[[i,j-1]]}$
$+su_{[[i+1,j-1]]}+(R/4)*h*((su_{[[i+1,j-1]]}-su_{[[i-1,j]]}))$
$+(r/4)*h*((su_{[[i+1,j-1]]}-su_{[[i-1,j-1]]}))$
$+(G_r*h*h*(u_{[[i,j-1]]}+u_{[[i,j]]}))-(2*M*h*h*su_{[[i+1,j-1]}););$
$su_{[[All,j]]}=\text{TriDiagonal }[V_a,sV_d,V_c,sb];];$

Tabulation:
Print[NumberForm[TableForm[N[Transpose[Chop[u]]],TableSpacing $->$\{0,2\}]]];
Print[NumberForm[TableForm[N[Transpose[Chop[su]]],TableSpacing $->$\{0,2\}]]];

Comparison$[n,m]$;

Print[TableForm[result, TableSpacing $->$\{0,2\}]];

Output: numerical solution for velocity profiles for $P_r = 0.733$

Table 2. Numerical solutions of velocity profiles for $P_r = 0.733$

t	y	Numerical solution
0.0975	0	1
0.0975	0.05	0.882349
0.0975	0.1	0.853675
0.0975	0.15	0.833967
0.0975	0.2	0.721045
0.0975	0.25	0.671107
0.0975	0.3	0.69668
0.0975	0.35	0.527745
0.0975	0.4	0.364691
0.0975	0.45	0.307724
0.0975	0.5	0.256872
0.0975	0.55	0.211999
0.0975	0.6	0.172823
0.0975	0.65	0.118938
0.0975	0.7	0.159839
0.0975	0.75	0.184943
0.0975	0.8	0.063609
0.0975	0.85	0.04516
0.0975	0.9	0.00889
0.0975	0.95	0.004079
0.0975	1	0

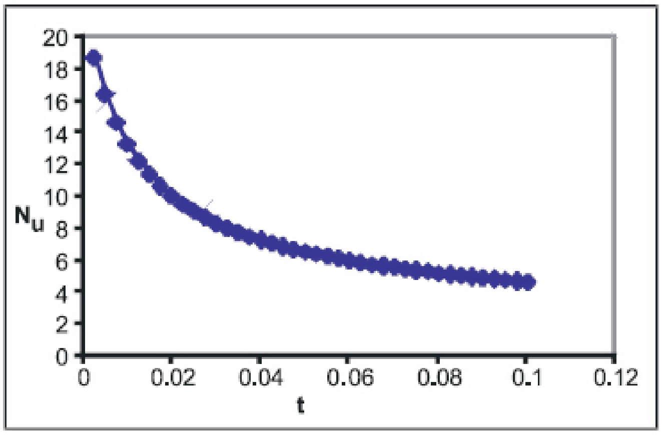

Figure 1. Rate of heat transfer

Table 3. Numerical solutions of rate of heat transfer

t	Numerical values of N_u
0.0025	11.69578
0.005	10.4067
0.0075	9.66101
0.01	8.29947
0.0125	7.21477
0.015	7.11348
0.0175	6.60449
0.02	5.991646
0.0225	5.468969
0.025	5.017448
0.0275	4.622961
0.03	4.274846
0.0325	3.964935
0.035	4.686876
0.0375	3.435664
0.04	3.207309
0.0425	2.998585
0.045	2.806863
0.0475	1.629974
0.05	1.466115

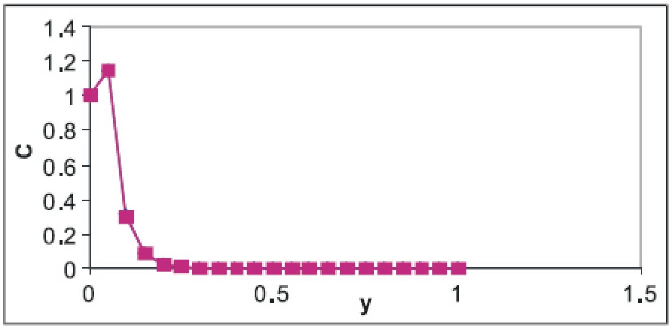

Figure 2. Numerical solutions of concentration for hydrogen $S_c = 0.22$

Table 4. Numerical solutions of concentration profiles for hydrogen $S_c = 0.22$

t	y	Numerical solution
0.0025	0	1
0.0025	0.05	1.042285
0.0025	0.1	0.543179
0.0025	0.15	0.283074
0.0025	0.2	0.147522
0.0025	0.25	0.07688
0.0025	0.3	0.040065
0.0025	0.35	0.02088
0.0025	0.4	0.010881
0.0025	0.45	0.005671
0.0025	0.5	0.002955
0.0025	0.55	0.00154
0.0025	0.6	0.000803
0.0025	0.65	0.000418
0.0025	0.7	0.000218
0.0025	0.75	0.000113
0.0025	0.8	5.89E-05
0.0025	0.85	3.02E-05
0.0025	0.9	1.49E-05
0.0025	0.95	6.10E-06
0.0025	1	0

Table 5. Rate of Mass transfer

S.No	S_c	S_h
1	0.22	0.69869
2	0.60	0.85989
3	0.78	0.36896

5. Results and Discussion

For the purpose of discussing the results some numerical solutions are obtained for non-dimensional temperature T, velocity u and concentration C. By using temperature the rate of heat transfer and by using concentration rate of mass transfer is obtained.
The temperature profiles for air ($P_r = 0.733$) for the case of injection are shown in Table 1. The numerical solutions for the case of injection for temperature have been shown in Table 1. It can be seen from the table that the transient

temperature decreases for the increase of y. Similarly temperature field due to variation in P_r for water, mercury etc has been found and observed that mercury has a stationary temperature. To save the space we are not giving full details as it is obvious to do. The concentration profiles for hydrogen $S_c = 0.22$ for the case of injection are shown in Figure 7. The numerical solutions for the case of injection for concentration have been shown in Table 4. It can be seen from the table as well as figure that the transient concentration profiles decreases for the increase of y. The concentration profiles due to variation of S_c for gases like oxygen, and water vapor has been found but not giving here due to almost similar calculations. It can be found that hydrogen can be used for maintaining effective concentration field. The transient velocity profiles for air ($P_r = 0.733$) for the case of injection are shown in Table 2. The numerical solutions for the case of injection for velocity have been shown in Table 2. It can be seen from the table that the transient velocity profiles decreases for the increase of y. While finding velocity profiles numerical values for G_r, G_m, M have been chosen suitably.

From the technological point of view, it is important to know the rate of heat transfer between the plate and the fluid. This can be found by using the non-dimensional quantity, the Nusselt number N_u. The Nusselt number is defined as negative gradient of the temperature. The numerical values of the Nusselt number against time t are shown in Figure 5 and Table 3. Figure 5 shows the heat transfer for different times. As t increases, the rate of heat transfer at the plate decreases gradually. Finally for mass transfer we need the negative gradient of concentration. This is denoted and defined as Schmidt number S_c. The numerical values of rate of mass transfer in terms of Sherwood number S_h are obtained and have been shown in Table 5. From this table it can be observed that rate of mass transfer first increases gradually and then decreases as per gradual increase and then decrease of the Schmidt number.

6. Conclusions

The main conclusions of this study are as follows:

(i) The transient temperature decreases for the case of air.
(ii) The transient concentration profiles decreases for the increase of y for the case of hydrogen.
(iii) The transient velocity profiles decreases for the increase of y for the case of air.
(iv) The rate of heat transfer at the plate decreases gradually.
(v) The rate of mass transfer first increases gradually and then decreases as per gradual increase and then decrease of the Schmidt number.

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