Selçuk J. Appl. Math.
Vol. 10. No. 2. pp. 81-94, 2009

Selçuk Journal of Applied Mathematics

Numerous Exact Solutions for the Dodd–Bullough–Mikhailov Equation by Some Different Methods

A. G. Davodi[1], D. D. Ganji[2], M. M. Alipour[3]

[1]Department of Civil Engineering, Shahrood University of Technology, Shahrood, Iran
e-mail: a.g.davodi@gmail.com
[2]Department of Mechanical Engineering, Babol University of Technology, Babol, Iran
e-mail: ddg_davood@yahoo.com
[3]Department of Mechanical Engineering, K.N Toosi University of Technology, Tehran, Iran
e-mail: mmalipour@yahoo.com

Received Date: March 16, 2009
Accepted Date: November 17, 2009

Abstract. In this work, we implement some analytical techniques such as Tan, Tanh, Extended Tanh and Sech methods for solving the nonlinear partial differential equation, which contain exponential terms; its name, Dodd–Bullough–Mikhailov (DBM) equation. These methods can be used as an alternative to obtain exact solutions of different types of differential equations which applied in engineering mathematics.

Key words: The Dodd–Bullough–Mikhailov (DBM) equation; Exp–Function, Tanh, Extended Tanh and Sech methods; Nonlinear Partial Differential equation.
2000 Mathematics Subject Classification: 35D10, 35D99, 35J70.

1.Introduction

The investigation of exact solutions of nonlinear evolution equations (NLEEs) plays an important role in the study of nonlinear physical phenomena. In the past several decades, many effective methods for obtaining exact solutions of NLEEs have been presented, such as Exp-function method [2-11], Tanh method [12-23], Sech method [24], Hirota direct method [25], rational hyperbolic method [26-28], He's Variational Iteration Method [29-33] and He's homotopy method [34-39]andsoon.
The class of equations, namely,

[2]Corresponding author

$$u_{xt} + f(u) = 0$$

Play a significant role in many scientific applications such as solid-state physics, nonlinear optics and quantum field theory.
The function $f(u)$takes many forms such as

$$f(u) = \begin{cases} \sin u \\ \sinh u \\ e^u \\ e^u + e^{-2u} \\ e^{-u} + e^{-2u} \end{cases} \tag{1}$$

that characterize the Sine–Gordon equation, sinh-Gordon equation, Liouville equation, Dodd–Bullough–Mikhailov equation (DBM), and the Tzitzeica–Dodd–Bullough (TDB) equation respectively.
In this work, we consider the Dodd–Bullough–Mikhailov (DBM) equation in the form:

$$u_{xt} + e^u + e^{-2u} = 0 \tag{2}$$

This equation appears in problems varying from fluid flow to quantum field theory. For solving this equation and for finding major solutions, we use the two transformations:
Transformation 1:

$$v(x,t) = e^{-u}, \quad u(x,t) = -\ln(v(x,t)) \tag{3}$$

Eq. (2) becomes a partial differential eqation, which reads

$$-vv_{xt} + v_x v_t + 1 + v^4 = 0 \tag{4}$$

To find the traveling wave solution of Eq. (4) we introduce the wave variable$\eta = \alpha x + \lambda t$ so that

$$-\alpha\lambda V V'' + \alpha\lambda V'^2 + V + V^4 = 0 \tag{5}$$

where prime denote the differential with respect to η.
Transformation 2:

$$v(x,t) = e^{u}, \quad u(x,t) = \ln(v(x,t)) \tag{6}$$

Eq. (2) becomes a partial differential eqation, which reads

$$vv_{xt} - v_x v_t + 1 + v^3 = 0 \tag{7}$$

To find the traveling wave solution of Eq. (7) we introduce the wave variable$\eta = \mu(x - \lambda t)$ so that

$$-\mu^2 \lambda V V''^2 \lambda V'^2 + 1 + V^3 = 0 \tag{8}$$

where prime denote the differential with respect to η.

2. Summary of methods

2.1. Tanh and Extended Tanh method

We consider nonlinear equation of form:

$$N\left(V, V', V''^3, \ldots\right) \tag{9}$$

In this section, we give a brief description of the extended tanh method as follows. We introduce the new independent variables:

$$Y = \begin{cases} \tanh(\eta) \\ \coth(\eta) \\ \tan(\eta) \\ \cot(\eta) \end{cases} \rightarrow Y' = \begin{cases} 1 - Y^2 \\ 1 - Y^2 \\ 1 + Y^2 \\ -1 - Y^2 \end{cases} \tag{10}$$

Since $Y = \tanh(\eta)$ or $\coth(\eta)$, repeatedly applying chain rule, we have:

$$\frac{d}{d\eta} = \frac{d}{dY}\frac{dY}{d\eta} = \left(1 - Y^2\right)\frac{d}{dY}$$

That leads to the change of derivatives

$$\begin{array}{l} \frac{d}{d\eta} = \left(1 - Y^2\right)\frac{d}{dY} \\ \frac{d^2}{d\eta^2} = \left(1 - Y^2\right)\frac{d}{dY}\left(\left(1 - Y^2\right)\frac{d}{dY}\right) \\ \frac{d^2}{d\eta^2} = \left(1 - Y^2\right)\frac{d}{dY}\left(\left(1 - Y^2\right)\frac{d}{dY}\left(\left(1 - Y^2\right)\frac{d}{dY}\right)\right) \end{array} \tag{12}$$

Similarly when$Y = \tan(\eta)$ or $Y = -\cot(\eta)$, we have:

$$\frac{d}{d\eta} = \frac{d}{dY}\frac{dY}{d\eta} = \left(1 + Y^2\right)\frac{d}{dY}$$

That leads to the change of derivatives

$$
\begin{aligned}
\frac{d}{d\eta} &= \left(1+Y^2\right)\frac{d}{dY} \\
\frac{d^2}{d\eta^2} &= \left(1+Y^2\right)\frac{d}{dY}\left(\left(1+Y^2\right)\frac{d}{dY}\right) \\
\frac{d^2}{d\eta^2} &= \left(1+Y^2\right)\frac{d}{dY}\left(\left(1+Y^2\right)\frac{d}{dY}\left(\left(1+Y^2\right)\frac{d}{dY}\right)\right)
\end{aligned} \tag{12}
$$

In the context of this method, many authors [12-17] used the ansatz

$$V\left(\eta\right) = \sum{}_{i=0}^{M} a_i Y^i\left(\eta\right) \tag{13}$$

In order to construct more general, it is reasonable to introduce the following ansatz [18-22]:

$$V\left(\eta\right) = \sum{}_{i=-M}^{M} a_i Y^i\left(\eta\right) \tag{14}$$

In which a_i and b_i ($i = 0$, 1…M) are all real constants to be determined later. The balancing number M is a positive integer, which can be determined by balancing the highest order derivative terms with highest power of nonlinear terms in Eq. (9). We substitute ansatz Eq. (13) or Eq. (14) into Eq. (9) and with aid of Eqs. (11-12) with computerized symbolic computation, equating to zero the coefficients of all power $Y^{\pm i}$ yields a set of algebraic equations for a_i and b_i .

2.2 The Sech method

We now describe the Sech method for the given partial differential equations. To use this method, we take following steps:
In a similar way of previous method, we consider nonlinear equation of form:

$$N\left(V, V', V''^3, \ldots\right) \tag{15}$$

We then introduce a new independent variable.

$$Y = \sec h(\eta),\ Y' = \frac{d}{d\eta}\sec h(\eta) \tag{16}$$

One computes:

$$
\begin{aligned}
Y' &= \frac{d}{d\eta}\sec h\left(\eta\right) = -\sec h\left(\eta\right)\ \tanh\left(\eta\right)\ = -\sec h\left(\eta\right)\ \sqrt{1-\sec h^2\left(\eta\right)} \\
Y'' &= \frac{d^2}{d\eta^2}\sec h\left(\eta\right) = -\sec h\left(\eta\right)\ \tanh^2\left(\eta\right)\ -\sec h^3\left(\eta\right) \\
&= \sec h\left(\eta\right)\ \left(1-\sec h^2\left(\eta\right)\ \right) - \sec h^3\left(\eta\right)
\end{aligned} \tag{17}
$$

Since$Y' = -Y\sqrt{(1-Y^2)}$, repeatedly applying chain rule, we have:

$$\frac{d}{d\eta}=\frac{d}{dY}\frac{dY}{d\eta}=-Y\sqrt{(1-Y^2)}\frac{d}{dY}$$

That leads to the change of derivates:

$$\begin{aligned}
&\tfrac{d}{d\eta}=-Y\sqrt{1-Y^2}\tfrac{d}{dY}\\
&\tfrac{d^2}{d\eta^2}=-Y\sqrt{1-Y^2}\left(-\sqrt{1-Y^2}\tfrac{d}{dY}+\tfrac{Y^2\frac{d}{dY}}{\sqrt{1-Y^2}}-Y\sqrt{1-Y^2}\tfrac{d^2}{dY^2}\right)\\
&\tfrac{d^2}{d\eta^2}=-Y\sqrt{1-Y^2}\Big(\left(1-6Y^2\right)\tfrac{d}{dY}+\left(3Y-6Y^3\right)\tfrac{d^2}{dY^2}\\
&\qquad\qquad+Y^2\left(1-Y^2\right)\tfrac{d^3}{dY^3}\Big)
\end{aligned}\tag{18}$$

Introducing the ansatz:

$$V(\eta)=S(\eta)=\sum{}_{i=0}^{M}a_iY^i(\eta)\tag{19}$$

where M is a positive integer parameter.
To determine the parameter M, we usually balance linear terms of highest order in the resulting equation with the highest order nonlinear terms. With M determined, equate the coefficients of powers of Y in the resulting equation. This will give a system of algebraic equation involving thea_i, $(i=0,\ldots,M)$.

3. New application of methods

Now, in this case we consider the Dodd–Bullough–Mikhailov (DBM) equation. For considering this equation, we solve this equation by some exact methods (Extended Tanh and Sech methods) which was explained in part 2 (summary of methods).

3.1 Using Tanh, Tan and Extended Tanh methods

In this case, we consider Eq. (8) using Extended Tanh method:
For determining values M in Eq. (13) and Eq. (14), we balance the linear term of the highest order in Eq. (8) with the highest order nonlinear term that yields $M=2$. Therefore, we have:

3.1.1. Tanh method

$$V(\eta)=a_0+a_1Y+a_2Y^2\tag{20}$$

where a_0, a_1 and a_2 will be determined and $Y(\eta)$ will satisfy Eq. (12).
Substituting Eq. (20) into Eq. (8) with the aid of Eq. (11), we get a system of algebraic equation, for $a_0,\ a_1,\ a_2,\ \mu$ andλ.
$Y^0=1-2\lambda\mu^2a_2a_0+a_0^3+\lambda\mu^2a_1^2$

$Y^1 = 2\lambda\mu^2 a_2 a_1 + 2\lambda\mu^2 a_1 a_0 + 3a_1 a_0^2$
$Y^2 = 3a_0 a_1^2 + 8\lambda\mu^2 a_2 a_0 + 3a_2 a_0^2 + 2\lambda\mu^2 a_2^2$
$Y^3 = 6a_0 a_1 a_2 + 2\lambda\mu^2 a_2 a_1 - 2\lambda\mu^2 a_1 a_0 + a_1^3$
$Y^4 = 3a_1^2 a_2 + 3a_0 a_2^2 - \lambda\mu^2 a_1^2 - 6\lambda\mu^2 a_2 a_0$
$Y^5 = -4\lambda\mu^2 a_2 a_1 + 3a_1 a_2^2$
$Y^6 = -2\lambda\mu^2 a_2^2 + a_2^3$
Solving the set of equation with the aid of Maple, we obtain:

$$\lambda = -\frac{3}{4\mu^2},\ a_0 = \frac{1}{2},\ a_1 = 0,\ a_2 = -\frac{3}{2} \tag{21}$$

Inserting these values into Eq. (20), we obtain

$$V(\eta) = \frac{1}{2} - \frac{3}{2}\tanh^2(\eta) \tag{22}$$

Substituting $\eta = \mu(x - \lambda t)$ into this result, we obtain:

$$v(x,t) = \frac{1}{2} - \frac{3}{2}\tanh^2(\mu(x - \lambda t)) \tag{23}$$

Moreover, from Eq. (21), we know $\lambda = -\frac{3}{4\mu^2}$ and then we have:

$$v(x,t) = \frac{1}{2} - \frac{3}{2}\tanh^2\left(\mu\left(x + \frac{3}{4\mu^2}t\right)\right) \tag{24}$$

From Eq. (6), we can obtain $u(x,t)$:

$$u(x,t) = \ln\left[\frac{1}{2} - \frac{3}{2}\tanh^2\left(\mu\left(x + \frac{3}{4\mu^2}t\right)\right)\right] \tag{25}$$

3.1.2. Tan method

Substituting Eq. (20) into Eq. (8) and with the aid of Eq. (12), we get a system of algebraic equation, for $a_0,\ a_1,\ a_2,\ \mu$andλ:
$Y^0 = 1 - 2\lambda\mu^2 a_2 a_0 + a_0^3 + \lambda\mu^2 a_1^2$
$Y^1 = 3a_1 a_0^2 + 2\lambda\mu^2 a_2 a_1 - 2\lambda\mu^2 a_1 a_0$
$Y^2 = 2\lambda\mu^2 a_2^2 + 3a_2 a_0^2 + 3a_0 a_1^2 - 8\lambda\mu^2 a_2 a_0$
$Y^3 = -2\lambda\mu^2 a_1 a_0 + a_1^3 - 2\lambda\mu^2 a_2 a_1 + 6a_0 a_1 a_2$
$Y^4 = 3a_1^2 a_2 + 3a_0 a_2^2 - \lambda\mu^2 a_1^2 - 6\lambda\mu^2 a_2 a_0$
$Y^5 = -4\lambda\mu^2 a_2 a_1 + 3a_1 a_2^2$
$Y^6 = -2\lambda\mu^2 a_2^2 + a_2^3$
Solving the set of equation with the aid of Maple, we obtain:

$$\lambda = \frac{3}{4\mu^2},\ a_0 = \frac{1}{2},\ a_1 = 0,\ a_2 = \frac{3}{2} \tag{26}$$

Inserting these values into "ansatz" Eq. (20), we obtain:

$$V(\eta) = \frac{1}{2} + \frac{3}{2} tan^2(\eta) \tag{27}$$

Substituting $\eta = \mu(x - \lambda t)$ into this result, we obtain:

$$v(x,t) = \frac{1}{2} + \frac{3}{2} tan^2(\mu(x - \lambda t)) \tag{28}$$

In addition, from Eq. (26) we know $\lambda = \frac{3}{4\mu^2}$ then we have:

$$v(x,t) = \frac{1}{2} + \frac{3}{2} tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right) \tag{29}$$

From Eq. (6), we can obtain $u(x,t)$:

$$u(x,t) = ln\left(\frac{1}{2} + \frac{3}{2} tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right)\right) \tag{30}$$

3.1.3. Extended Tanh method

In this case, we consider Eq. (8) using Extended Tanh method:

$$V(\eta) = a_{-2}Y^{-2} + a_{-1}Y^{-1} + a_0 + a_1Y + a_2Y^2 \tag{31}$$

Substituting Eq. (31) into Eq. (8) with the aid of Eq. (12), we get a system of algebraic equation, for $a_{-2},\ a_{-1},\ a_0,\ a_1,\ a_2,\ \mu$ andλ:

$\frac{1}{Y^6} = a_{-2}^3 - 2\lambda\mu^2 a_{-2}^2$

$\frac{1}{Y^5} = -4\lambda\mu^2 a_{-1}a_{-2} + 3a_{-1}a_{-2}^2$

$\frac{1}{Y^4} = 3a_0a_{-2}^2 - 6\lambda\mu^2 a_{-2}a_0 - c\mu^2 a_{-1}^2 + 3a_{-1}^2 a_{-2}$

$\frac{1}{Y^3} = 3a_1a_{-2}^2 + 6a_0a_{-2}a_{-1} + 2\lambda\mu^2 a_{-1}a_{-2} - 10\lambda\mu^2 a_{-2}a_1 - 2\lambda\mu^2 a_{-1}a_1 + a_{-1}^3$

$\frac{1}{Y^2} = -16\lambda\mu^2 a_2a_{-2} + 6a_1a_{-2}a_{-1} + 8\lambda\mu^2 a_{-2}a_0 + 3a_0a_{-1}^2 - 4\lambda\mu^2 a_1a_{-1} + 3a_2a_{-2}^2 + 3a_{-2}a_0^2 + 2\lambda\mu^2 a_{-2}^2$

$Y^0 = 32\lambda\mu^2 a_2a_{-2} + 8\lambda\mu^2 a_1a_{-1} - 2\lambda\mu^2 a_2a_0 + 1 + 3a_1^2 a_{-2} + \lambda\mu^2 a_1^1 + a_0^3 + 3a_2a_{-1}^2 + 6a_0a_2a_{-2} + \lambda\mu^2 a_{-1}^2 + 6a_0a_1a_{-1} - 2\lambda\mu^2 a_{-2}a_0$

$Y^1 = 6a_1a_2a_{-2} + 3a_1^2a_{-1} + 18\lambda\mu^2a_2a_{-1} - 8\lambda\mu^2a_{-2}a_1 + 3a_1a_0^2 + 2\lambda\mu^2a_2a_1 + 6a_0a_2a_{-1} + 2\lambda\mu^2a_1a_0$
$Y^2 = -16\lambda\mu^2a_2a_{-2} + 2\lambda\mu^2a_2^2 + 6a_1a_2a_{-1} - 4\lambda\mu^2a_1a_{-1} + 3a_2a_0^2 + 3a_2^2a_{-2} + 8\lambda\mu^2a_2a_0 + 3a_0a_1^2$
$Y^3 = a_1^3 - 2\lambda\mu^2a_1a_0 + 2\lambda\mu^2a_2a_1 - 10\lambda\mu^2a_2a_{-1} + 3a_2^2a_{-1} + 6a_0a_1a_2$
$Y^4 = -6\lambda\mu^2a_2a_0 + 3a_1^2a_2 + 3a_0a_2^2 - c\mu^2a_1^2$
$Y^5 = 3a_1a_2^2 - 4\lambda\mu^2a_2a_1$
$Y^6 = a_2^3 - 2\lambda\mu^2a_2^2$
Solving the set of equation with the aid of Maple, we can distinguish different cases namely:
Case 1:

$$\lambda = -\frac{3}{4\mu^2},\ a_{-2} = -\frac{3}{2},\ a_{-1} = 0,\ a_0 = \frac{1}{2},\ a_1 = 0,\ a_2 = 0 \tag{32}$$

Inserting these values into "ansatz" Eq. (31), we obtain:

$$V(\eta) = \frac{1}{2} - \frac{3}{2\tanh^2(\eta)} \tag{33}$$

Substituting $\eta = \mu(x - \lambda t)$ into this result, we obtain:

$$v\left(x,t\right) = \frac{1}{2} + \frac{3}{2}tan^2\left(\mu\left(x - \lambda t\right)\right) \tag{34}$$

In addition, from Eq. (32), we know $\lambda = -\frac{3}{4\mu^2}$, and then we have:

$$v\left(x,t\right) = \frac{1}{2} + \frac{3}{2}tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right) \tag{35}$$

From Eq. (6), we can obtain $u\left(x,t\right)$:

$$u\left(x,t\right) = \ln\left(\frac{1}{2} + \frac{3}{2}\tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right)\right) \tag{36}$$

Case 2:

$$\lambda = -\frac{3}{4\mu^2},\ a_{-2} = 0,\ a_{-1} = 0,\ a_0 = \frac{1}{2},\ a_1 = 0,\ a_2 = -\frac{3}{2} \tag{37}$$

Inserting these values into "ansatz" Eq. (31), we obtain:

$$V(\eta) = \frac{1}{2} - \frac{3}{2}\tanh^2(\eta) \tag{38}$$

Substituting $\eta = \mu(x - \lambda t)$ into these results, we obtain:

$$v\left(x,t\right) = \frac{1}{2} + \frac{3}{2} tan^2\left(\mu\left(x - \lambda t\right)\right) \tag{39}$$

In addition, from Eq. (37), we know$\lambda = -\frac{3}{4\mu^2}$, and then we have:

$$v\left(x,t\right) = \frac{1}{2} + \frac{3}{2} tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right) \tag{40}$$

From Eq. (6), we can obtain$u\left(x,t\right)$:

$$u\left(x,t\right) = \ln\left[\frac{1}{2} + \frac{3}{2} tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right)\right] \tag{41}$$

Case 3:

$$\lambda = -\frac{3}{16\mu^2},\ a_{-2} = -\frac{3}{8},\ a_{-1} = 0,\ a_0 = \frac{1}{4},\ a_1 = 0,\ a_2 = -\frac{3}{8} \tag{42}$$

Inserting these values into "ansatz" Eq. (31), we obtain:

$$V\left(\eta\right) = -\frac{1}{4} - \frac{3}{8}\tanh^2\left(\eta\right) - \frac{3}{8}\tanh^{-2}\left(\eta\right) \tag{43}$$

Substituting $\eta = \mu(x - \lambda t)$ into these results, we obtain:

$$v\left(x,t\right) = -\frac{3}{8}\tanh^2\left(\mu(x - \lambda t)\right) - \frac{1}{4} - \frac{3}{8}\tanh^{-2}\left(\mu(x - \lambda t)\right) \tag{44}$$

In addition, from Eq. (42), we know $\lambda = -\frac{3}{16\mu^2}$, and then we have:

$$v\left(x,t\right) = -\frac{3}{8}\tanh^2\left(\mu(x + \frac{3}{16\mu^2}t)\right) - \frac{1}{4} - \frac{3}{8}\tanh^{-2}\left(\mu(x + \frac{3}{16\mu^2}t)\right) \tag{45}$$

From Eq. (6), we obtain $u\left(x,t\right)$:

$$u\left(x,t\right) = \ln\left[-\frac{3}{8}\tanh^2\left(\mu(x + \frac{3}{16\mu^2}t)\right) - \frac{1}{4} - \frac{3}{8}\tanh^{-2}\left(\mu(x + \frac{3}{16\mu^2}t)\right)\right] \tag{46}$$

And at the same we can obtain three solutions using Extended tan method:

$$u\left(x,t\right) = \ln\left(\frac{1}{2} + \frac{3}{2\tan^2\left(\mu\left(x - \frac{3}{4\mu^2}t\right)\right)}\right) \tag{47}$$

$$u\left(x,t\right)=\ln\left[\frac{1}{2}+\frac{3}{2}\tan^{2}\left(\mu\left(x-\frac{3}{4\mu^{2}}t\right)\right)\right] \tag{48}$$

$$u\left(x,t\right)=\ln\left[\frac{3}{8}\tan^{2}\left(\mu(x-\frac{3}{16\mu^{2}}t)\right)-\frac{1}{4}+\frac{3}{8}\tan^{-2}\left(\mu(x-\frac{3}{16\mu^{2}}t)\right)\right] \tag{49}$$

3.2 Using Sec and Sech method

In this case, we consider DBM equation using Sech method that was explains above:

$$-\mu^{2}\lambda VV''^{2}\lambda V'^{2}+1+V^{3}=0 \tag{50}$$

For determining values M in Eq. (19), we balance the linear term of the highest order in Eq. (50) with the highest order nonlinear term that yields M=2. Therefore, we have:

$$V(\eta)=a_{0}+a_{1}Y+a_{2}Y^{2} \tag{51}$$

where a_0, a_1 and a_2 will be determined.
Substituting Eq. (51) into Eq. (50), we get a system of algebraic equation, for
$Y^0=1+a_0^3$
$Y^1=3a_0^2a_1-\lambda\mu^2a_1a_0$
$Y^2=3a_0a_1^2+3a_0^2a_2-4\lambda\mu^2a_2a_0$
$Y^3=2\lambda\mu^2a_1a_0+6a_0a_1a_2-\lambda\mu^2a_1a_2+a_1^3$
$Y^4=3a_0a_2^2+3a_1^2a_2+6\lambda\mu^2a_2a_0+\mu^2a_1^2$
$Y^5=4\lambda\mu^2a_1a_2+3a_1a_2^2$
$Y^6=a_2^3+2\lambda\mu^2a_2^2$
Solving the set of equation, we obtain:

$$\lambda=\frac{-3}{4\mu^{2}},\ a_{0}=-1,\ a_{1}=0,\ a_{2}=\frac{3}{2} \tag{52}$$

Inserting these values into "ansatz" Eq. (51), we obtain:

$$V(\eta)=-1+\frac{3}{2}\sec h^{2}(\eta) \tag{53}$$

Substituting $\eta=\mu(x-\lambda t)$ into this result, we obtain:

$$v(x,t)=-1+\frac{3}{2}\sec h^{2}(\mu(x-\lambda t)) \tag{54}$$

In addition, from Eq. (52), we know, $\lambda = \frac{-3}{4\mu^2}$ and then we have:

$$v(x,t) = -1 + \frac{3}{2}\sec h^2(\mu(x + \frac{3}{4\mu^2}t)) \tag{55}$$

From Eq. (6), we obtain $u\,(x,t)$:

$$u(x,t) = \ln\left[-1 + \frac{3}{2}\sec h^2(\mu(x + \frac{3}{4\mu^2}t))\right] \tag{56}$$

And at the same we can obtain a solution using sec method:

$$u(x,t) = \ln\left[-1 + \frac{3}{2}\sec^2(\mu(x - \frac{3}{4\mu^2}t))\right] \tag{57}$$

4. Discussion and conclusion

A comparative study between the *tan* and *tanh* method, extended *tan and tanh* method and the sech method was present. The Dodd–Bullough–Mikhailov equation illustrates and explores the power of these methods. Many types of exact solutions with distinct physical structures have been found.
The tanh and the tan method are used for finding the Dodd–Bullough–Mikhailov equation's solutions. These method can be easily extended to other nonlinear evaluation equations of any order with the help of symbolic computation (Mathematica or Matlab, Maple, etc.). The technique is a straightforward solution to find a closed form. The tanh strength lies in its ease of use and the possibility of using it as a tool to acquire approximate solutions.
For finding some better result in tanh or tan method, we use the Extended tanh method. In this paper by using this method, we find some more solutions besides tan method for the Dodd–Bullough–Mikhailov equation.
In addition we used Sech method finding the Dodd–Bullough–Mikhailov equation's solution. The Sech method is useful and simple method to solve nonlinear equation. These results could be used as a starting point for other numerical procedures to achieve much better results.

References:

1. A. M. Wazwaz, The tanh method solitons and periodic solutions for the Dodd–Bullough–Mikhailov and the Tzitzeica–Dodd–Bullough equations, Chaos Solitons Fractals 25 (2005) 55–63.
2. J.-H. He, Xu.-H. Wu, Exp-function method for nonlinear wave equations, Chaos Solitons Fractals 30 (2006) 700–708.
3. M. A. Abdou, A. A. Soliman, S. T. El-Basyony, New application of Exp-function method for improved Boussinesq equation, Physics Letters A 369 (2007) 469–475.

4. Xu-Hong (Benn) Wu, Ji-Huan He, Solitary solutions, periodic solutions and compacton-like solutions using the Exp-function method, Computers and Mathematics with Applications 54 (2007) 966–986.
5. E. Yusufoglu, A. Bekir Symbolic computation and new families of exact travelling solutions for the Kawahara and modified Kawahara equations, Computers and Mathematics with Applications 55 (2008) 1113–1121.
6. Ji-Huan He, M. A. Abdou, New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos, Solitons and Fractals 34 (2007) 1421–1429
7. Abdul-Majid Wazwaz, Solitary wave solutions of the generalized shallow water wave (GSWW) equation by Hirota's method, tanh-coth method and Exp-function method, Appl. Math. Comput. 202 (1) (2008) 275-286.
8. A. Bekir, A. Boz, Application of Exp-function method for (2 + 1)-dimensional nonlinear evolution equations, Chaos, Solitons & Fractals, 40 (1) (2009) 458–465.
9. D. D. Ganji, A. G. Davodi, Y. A. Geraily, New exact solutions for seventh-order Sawada-Kotera-Ito, Lax and Kaup-Kupershmidt equations using EXP-Function mehod Mathematical Methods in the Applied Sciences, Doi: 10.1002/mma.1160.
10. Amin G. Davodi, D. D. Ganji, Arash G. Davodi, A. Asgari, Finding general and explicit solutions (2 +1) dimensional Broer-Kaup-Kupershmidt systemnonlinear equation by Exp-Function method, Applied Mathematics and Computation, Doi: 10.1016/j.amc.2009.05.069.
11. M. M. Alipour, D. D. Ganji, A. G. Davodi, An application of Exp–Function method to The Generalized Burger's–Huxley equation, SELCUK JOURNAL OF APPLIED MATHEMATICS, Vol. 10. No. 1. (2009) 121-133.
12. G. Domairry, Amin. G. Davodi, Arash. G. Davodi Solutions for the Double Sine-Gordon equations by Exp-function method, Tanh and Extended Tanh methods, Numerical Method For Partial Differential Equation, Doi: 10.1002/num.20440.
13. Abdul-Majid Wazwaz, Multiple-soliton solutions for the Lax–Kadomtsev–Petviashvili (Lax–KP) equation, Applied Mathematics and Computation 201 (2008) 168–174.
14. S. A. El-Wakil, M. A. Abdou, A. Hendi, New periodic and soliton solutions of nonlinear evolution equations, Applied Mathematics and Computation 197 (2008) 497–506.
15. Huiqun Zhang, New exact solutions for two generalized Hirota–Satsuma coupled KdV systems, Communications in Nonlinear Science and Numerical Simulation 12 (2007) 1120–1127.
16. Abdul-Majid Wazwaz, New kinks and solitons solutions to the (2 + 1)-dimensional Konopelchenko–Dubrovsky equation, Mathematical and Computer Modelling 45 (2007) 473–479.
17. E. M. E. Zayed, H. A. Zedan, K. A. Gepreel, On the solitary wave solutions for nonlinear Hirota–Satsuma coupled KdV of equations Chaos, Solitons and Fractals 22 (2004) 285–303.
18. M. A. Abdou, A. A. Soliman, Modified extended tanh-function method and its application on nonlinear physical equations, Physics Letters A 353 (2006) 487–492.
19. E. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A 277 (2000) 212.
20. M. A. Abdou, A. A. Soliman, Modified extended tanh-function method and its application on nonlinear physical equations, Physics Letters A 353 (2006) 487–492.

21. Abdul-Majid Wazwaz, The extended tanh method for the Zakharov–Kuznetsov (ZK) equation, the modified ZK equation, and its generalized forms, Communications in Nonlinear Science and Numerical Simulation 13 (2008) 1039–1047.
22. A. H. A. Ali, The modified extended tanh-function method for solving coupled MKdV and coupled Hirota–Satsuma coupled KdV equations, Physics Letters A 363 (2007) 420–425.
23. W. Malfliet, W. Hereman, The tanh method: I. Exact solutions of nonlinear evolution and wave equations, Physica. Scripta 54 (1996) 563–568.
24. Abdul-Majid Wazwaz, The tanh–coth and the sech methods for exact solutions of the Jaulent–Miodek equation, Physics Letters A 366 (2007) 85–90.
25. A. M. Wazwaz, The Hirota's bilinear method and the tanh–coth method for multiple-soliton solutions of the Sawada–Kotera–Kadomtsev–Petviashvili equation, Applied Mathematics and Computation 200 (2008) 160–166.
26. A. M. Wazwaz, New travelling wave solutions to the Boussinesq and the Klein–Gordon equations, Communications in Nonlinear Science and Numerical Simulation 13 (2008) 889–901.
27. A. M. Wazwaz, New solitary wave solutions to the modified forms of Degasperis–Procesi and Camassa–Holm equations, Applied Mathematics and Computation 186 (2007) 130–141.
28. A. M. Wazwaz, The tanh–coth method for new compactons and solitons solutions for the K(n,n) and the K(n + 1,n + 1) equations, Applied Mathematics and Computation 188 (2007) 1930–1940.
29. A. M. Wazwaz, The variational iteration method for solving two forms of Blasius equation on a half-infinite domain, Applied Mathematics and Computation 188 (2007) 485–491.
30. A. A. Soliman, M. A. Abdou, Numerical solutions of nonlinear evolution equations using variational iteration method, Journal of Computational and Applied Mathematics 207 (2007) 111 – 120.
31. SHA. Hashemi. Kachapi, D. D. Ganji , A. G. Davodi, S. M. Varedi, Periodic Solution for Strongly Nonlinear Vibration Systems by He's Variational iteration method, Mathematical methods in applied science, Doi: 10.1002/mma.1135.
32. M. Esmaeilpour, D. D. Ganji, A. G. Davodi, N. Sadoughi, Application of He's methods for laminar flow in a porous-saturated pipe, Mathematical Methods in the Applied Sciences, DOI: 10.1002/mma.1240.
33. D. D. Ganji_, M. Nourollahi, E. Mohseni, Application of He's methods to nonlinear chemistry problems, Computers and Mathematics with Applications 54 (2007) 1122–1132.
34. A. Rajabi, D. D. Ganji, H. Taherian, Application of homotopy perturbation method in nonlinear heat conduction and convection equations, Physics Letters A 360 (2007) 570–573.
35. A. Sadighi, D. D. Ganji, Analytic treatment of linear and nonlinear Schrödinger equations: A study with homotopy-perturbation and Adomian decomposition methods, Physics Letters A, 372 (2008) 465–469.
36. D. D. Ganji, The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer, Physics Letters A, 355 (2006) 337–341.

37. M. Rafei, H. Daniali, D. D. Ganji, H. Pashaei, Solution of the prey and predator problem by homotopy perturbation method, Applied Mathematics and Computation 188 (2007) 1419–1425.
38. D. D. Ganji, G. A. Afrouzi, H. Hosseinzadeh, R. A. Talarposhti, Application of homotopy-perturbation method to the second kind of nonlinear integral equations, Physics Letters A 371 (2007) 20–25.
39. D. D. Ganji, M. J. Hosseini, J. Shayegh, Some nonlinear heat transfer equations solved by three approximate methods, International Communications in Heat and Mass Transfer 34 (2007) 1003–1016.